Do you like working with your hands to solve problems? If so, you might want to become a skilled trade worker, such as a master plumber.

To calculate the cost of parts and labor, plumbers use basic mathematical formulas. For example, some plumbers might calculate the cost of a new sewer line with a formula like the following:

\[
\text{cost of installed line} = \frac{\text{cost of pipe}}{3} \times 49 + \frac{\text{cost of pipe fittings}}{2}
\]
Vocabulary

Choose the best term from the list to complete each sentence.

1. To find the sum of two numbers, you should __?__.
2. Fractions are written as a __?__ over a __?__.
3. In the equation $4 \cdot 3 = 12$, 12 is the __?__.
4. The __?__ of 18 and 10 is 8.
5. The numbers 18, 27, and 72 are __?__ of 9.

Write and Read Decimals

Write each decimal in word form.

6. 0.5
7. 2.78
8. 0.125
9. 12.8
10. 125.49
11. 8.024

Multiples

List the first four multiples of each number.

12. 6
13. 8
14. 5
15. 12
16. 7
17. 20
18. 14
19. 9

Evaluate Expressions

Evaluate each expression for the given value of the variable.

20. $y + 4.3$ for $y = 3.2$
21. $\frac{x}{5}$ for $x = 6.4$
22. $3c$ for $c = 0.75$
23. $a + 4 \div 8$ for $a = 3.75$
24. $27.8 - d$ for $d = 9.25$
25. $2.5b$ for $b = 8.4$

Factors

Find all the whole-number factors of each number.

26. 8
27. 12
28. 24
29. 30
30. 45
31. 52
32. 75
33. 150
Where You’ve Been

Previously, you
- identified a number as prime or composite.
- identified common factors of a set of whole numbers.
- generated equivalent fractions.
- compared two fractions with common denominators.

In This Chapter

You will study
- writing the prime factorization of a number.
- finding the greatest common factor (GCF) of a set of whole numbers.
- generating equivalent forms of numbers, including whole numbers, fractions, and decimals.
- comparing and ordering fractions, decimals, and whole numbers.

Key Vocabulary/Vocabulario

<table>
<thead>
<tr>
<th>English</th>
<th>Spanish</th>
</tr>
</thead>
<tbody>
<tr>
<td>common denominator</td>
<td>común denominador</td>
</tr>
<tr>
<td>composite number</td>
<td>número compuesto</td>
</tr>
<tr>
<td>equivalent fractions</td>
<td>fracciones equivalentes</td>
</tr>
<tr>
<td>factor</td>
<td>factor</td>
</tr>
<tr>
<td>greatest common factor</td>
<td>máximo común divisor (MCD)</td>
</tr>
<tr>
<td>(GCF)</td>
<td></td>
</tr>
<tr>
<td>improper fraction</td>
<td>fracción impropia</td>
</tr>
<tr>
<td>prime factorization</td>
<td>factorización prima</td>
</tr>
<tr>
<td>prime number</td>
<td>número primo</td>
</tr>
<tr>
<td>terminating decimal</td>
<td>decimal cerrado</td>
</tr>
</tbody>
</table>

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The word *equivalent* means “equal in value.” What do you think *equivalent fractions* are?

2. To *terminate* something means to bring it to an end. If a decimal is a *terminating decimal*, what do you think happens to it? Explain.

3. When people have something in *common*, they have something that they share. What do you think *common denominators* share?

4. If something is *improper*, it is not right. In fractions, it is *improper* to have the numerator be greater than the denominator. How would you expect an *improper fraction* to look?

You can use the skills learned in this chapter
- to double or halve recipes when cooking.
- to add together fractions when determining volume in a science class.
Reading Strategy: Read a Lesson for Understanding

Reading ahead will prepare you for new ideas and concepts presented in class. As you read a lesson, make notes. Write down the main points of the lesson, math terms that you do not understand, examples that need more explanation, and questions you can ask during class.

Learn to solve equations involving decimals. The objective tells you the main idea of the lesson. Work through the examples and write down any questions you have.

Solving One-Step Equations with Decimals

Solve each equation. Check your answer.

\[
g - 3.1 = 4.5 \\
\text{Add 3.1 to both sides to undo the subtraction.}
\]

Check

\[
g - 3.1 = 4.5
\]

Substitute 7.6 for \( g \) in the equation.

\[
7.6 - 3.1 = 4.5
\]

\[
4.5 = 4.5 \checkmark
\]

Questions:

- How do I know what operation to use?
- What should I do if I check my answer and the two sides are not equal?

Try This

Read Lesson 4-1 before your next class and answer the following questions.

1. What is the objective of the lesson?
2. Are there new vocabulary terms, formulas, or symbols? If so, what are they?
Learn to use divisibility rules.

Vocabulary
- divisible
- composite number
- prime number

This year, 42 girls signed up to play basketball for the Junior Girls League, which has 6 teams. To find whether each team can have the same number of girls, decide if 42 is divisible by 6.

A number is divisible by another number if the quotient is a whole number with no remainder.

\[
42 \div 6 = 7 \longleftarrow \text{Quotient}
\]

Since there is no remainder, 42 is divisible by 6. The Junior Girls League can have 6 teams with 7 girls each.

### Divisibility Rules

<table>
<thead>
<tr>
<th>A number is divisible by…</th>
<th>Divisible</th>
<th>Not Divisible</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 if the last digit is even (0, 2, 4, 6, or 8).</td>
<td>3,978</td>
<td>4,975</td>
</tr>
<tr>
<td>3 if the sum of the digits is divisible by 3.</td>
<td>315</td>
<td>139</td>
</tr>
<tr>
<td>4 if the last two digits form a number divisible by 4.</td>
<td>8,512</td>
<td>7,518</td>
</tr>
<tr>
<td>5 if the last digit is 0 or 5.</td>
<td>14,975</td>
<td>10,978</td>
</tr>
<tr>
<td>6 if the number is divisible by both 2 and 3.</td>
<td>48</td>
<td>20</td>
</tr>
<tr>
<td>9 if the sum of the digits is divisible by 9.</td>
<td>711</td>
<td>93</td>
</tr>
<tr>
<td>10 if the last digit is 0.</td>
<td>15,990</td>
<td>10,536</td>
</tr>
</tbody>
</table>

### Example 1

#### Checking Divisibility

**A.** Tell whether 610 is divisible by 2, 3, 4, and 5.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td><em>The last digit, 0, is even.</em></td>
</tr>
<tr>
<td>3</td>
<td><em>The sum of the digits is 6 + 1 + 0 = 7. 7 is not divisible by 3.</em></td>
</tr>
<tr>
<td>4</td>
<td><em>The last two digits form the number 10. 10 is not divisible by 4.</em></td>
</tr>
<tr>
<td>5</td>
<td><em>The last digit is 0.</em></td>
</tr>
</tbody>
</table>

So 610 is divisible by 2 and 5.
B Tell whether 387 is divisible by 6, 9, and 10.

<table>
<thead>
<tr>
<th>Number</th>
<th>Condition</th>
<th>Divisibility</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>The last digit, 7, is odd, so 387 is not divisible by 2.</td>
<td>Not divisible</td>
</tr>
<tr>
<td>9</td>
<td>The sum of the digits is $3 + 8 + 7 = 18$. 18 is divisible by 9.</td>
<td>Divisible</td>
</tr>
<tr>
<td>10</td>
<td>The last digit is 7, not 0.</td>
<td>Not divisible</td>
</tr>
</tbody>
</table>

So 387 is divisible by 9.

Any number greater than 1 is divisible by at least two numbers—1 and the number itself. Numbers that are divisible by more than two numbers are called **composite numbers**.

A **prime number** is divisible by only the numbers 1 and itself. For example, 11 is a prime number because it is divisible by only 1 and 11. The numbers 0 and 1 are neither prime nor composite.

**Example 2**

**Identifying Prime and Composite Numbers**

Tell whether each number is prime or composite.

**A 45**
- Divisible by 1, 3, 5, 9, 15, 45
- Composite

**B 13**
- Divisible by 1, 13
- Prime

**C 19**
- Divisible by 1, 19
- Prime

**D 49**
- Divisible by 1, 7, 49
- Composite

The prime numbers from 1 through 50 are highlighted below.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>12</td>
<td>13</td>
<td>14</td>
<td>15</td>
<td>16</td>
<td>17</td>
<td>18</td>
<td>19</td>
<td>20</td>
</tr>
<tr>
<td>21</td>
<td>22</td>
<td>23</td>
<td>24</td>
<td>25</td>
<td>26</td>
<td>27</td>
<td>28</td>
<td>29</td>
<td>30</td>
</tr>
<tr>
<td>31</td>
<td>32</td>
<td>33</td>
<td>34</td>
<td>35</td>
<td>36</td>
<td>37</td>
<td>38</td>
<td>39</td>
<td>40</td>
</tr>
<tr>
<td>41</td>
<td>42</td>
<td>43</td>
<td>44</td>
<td>45</td>
<td>46</td>
<td>47</td>
<td>48</td>
<td>49</td>
<td>50</td>
</tr>
</tbody>
</table>

**Think and Discuss**

1. **Tell** which whole numbers are divisible by 1.
2. **Explain** how you know that 87 is a composite number.
3. **Tell** how the divisibility rules help you identify composite numbers.
4-1 Exercises

**GUIDED PRACTICE**

Tell whether each number is divisible by 2, 3, 4, 5, 6, 9, and 10.

1. 508
2. 432
3. 247
4. 189

Tell whether each number is prime or composite.

5. 75
6. 17
7. 27
8. 63
9. 72
10. 83
11. 99
12. 199

**INDEPENDENT PRACTICE**

Tell whether each number is divisible by 2, 3, 4, 5, 6, 9, and 10.

13. 741
14. 810
15. 675
16. 480
17. 908
18. 146
19. 514
20. 405

Tell whether each number is prime or composite.

21. 34
22. 29
23. 61
24. 81
25. 51
26. 23
27. 97
28. 93
29. 77
30. 41
31. 67
32. 39

**PRACTICE AND PROBLEM SOLVING**

Copy and complete the table. Write *yes* if the number is divisible by the given number. Write *no* if it is not.

<table>
<thead>
<tr>
<th></th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>33.</td>
<td>677</td>
<td>no</td>
<td></td>
<td></td>
<td>no</td>
<td></td>
<td>no</td>
</tr>
<tr>
<td>34.</td>
<td>290</td>
<td>yes</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>35.</td>
<td>1,744</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>36.</td>
<td>12,180</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Tell whether each statement is true or false. Explain your answers.

37. All even numbers are divisible by 2.
38. All odd numbers are divisible by 3.
39. Some even numbers are divisible by 5.
40. All odd numbers are prime.

Replace each box with a digit that will make the number divisible by 3.

41. 74 [ ]
42. 8,10 [ ]
43. 3, [ ] 41
44. [ ], 335
45. 67, [ ] 11
46. 10,0 [ ] 1
47. Make a table that shows the prime numbers from 50 to 100.

48. **Astronomy** Earth has a diameter of 7,926 miles. Tell whether this number is divisible by 2, 3, 4, 5, 6, 9, and 10.

49. On which of the bridges in the table could a light fixture be placed every 6 meters so that the first light is at the beginning of the bridge and the last light is at the end of the bridge? Explain.

<table>
<thead>
<tr>
<th>Name and State</th>
<th>Length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Verrazano Narrows, NY</td>
<td>1,298</td>
</tr>
<tr>
<td>Golden Gate, CA</td>
<td>1,280</td>
</tr>
<tr>
<td>Mackinac Straits, MI</td>
<td>1,158</td>
</tr>
<tr>
<td>George Washington, NY</td>
<td>1,067</td>
</tr>
</tbody>
</table>

50. **Critical Thinking** A number is between 80 and 100 and is divisible by both 5 and 6. What is the number?

51. **Choose a Strategy** Find the greatest four-digit number that is divisible by 1, 2, 3, and 4.

52. **What's the Error?** To find whether 3,463 is divisible by 4, a student added the digits. The sum, 16, is divisible by 4, so the student stated that 3,463 is divisible by 4. Explain the error.

53. **Write About It** If a number is divisible by both 4 and 9, by what other numbers is it divisible? Explain.

54. **Challenge** Find a number that is divisible by 2, 3, 4, 5, 6, and 10, but not 9.

55. **Multiple Choice** ___?___ numbers are divisible by more than two numbers.
   - Whole
   - Prime
   - Equivalent
   - Composite

56. **Short Response** What is the least three-digit number that is divisible by both 5 and 9? Show your work.

Use the pattern to write the first five terms of each sequence. (Lesson 1-7)

57. Start with 7; add 4.  
58. Start with 78; subtract 9.  
59. Start with 6; multiply by 5.

Evaluate each expression for the given value of the variable. (Lesson 2-1)

60. $2x + 28$ for $x = 4$  
61. $x + 18$ for $x = 12$  
62. $\frac{x}{3}$ for $x = 25$
Explore Factors

You can use graph paper or unit cubes to model factors of a number and determine whether the number is a prime number or a composite number.

**Activity**

Use graph paper to show the different ways the number 16 can be modeled.

1. The number 16 can be modeled by drawing a rectangle 2 units wide and 8 units long. The dimensions, 2 and 8, are factors of 16. This means that $2 \times 8 = 16$.

What other ways can 16 be modeled?

A rectangle 1 unit wide and 16 units long and a 4-unit-by-4 unit square can also model 16.

The factors of 16 are 1, 2, 4, 8, and 16. Because you can model 16 in more than one way, 16 is a composite number.

Use graph paper to show the different ways the number 3 can be modeled.

2. The number 3 can be modeled by drawing a rectangle 1 unit wide and 3 units long. The dimensions, 1 and 3, are factors of 3. This means that $1 \times 3 = 3$.

Because 3 cannot be modeled any other way, 3 is a prime number.

**Think and Discuss**

1. How can you use the rules of divisibility to determine whether there is more than one way to model a number?

2. Find the factors of 2. Is 2 prime or composite? Explain.

**Try This**

1. Use graph paper to model two prime numbers and two composite numbers. Find their factors.
Learn to write prime factorizations of composite numbers.

Whole numbers that are multiplied to find a product are called factors of that product. A number is divisible by its factors.

Finding Factors

List all of the factors of each number.

A 18

Begin listing factors in pairs.

18 = 1 \cdot 18
18 = 2 \cdot 9
18 = 3 \cdot 6

18 = 6 \cdot 3

The factors of 18 are 1, 2, 3, 6, 9, and 18.

B 13

13 = 1 \cdot 13

The factors of 13 are 1 and 13.

You can use factors to write a number in different ways.

<table>
<thead>
<tr>
<th>Factorization of 12</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 \cdot 12</td>
</tr>
<tr>
<td>2 \cdot 6</td>
</tr>
<tr>
<td>3 \cdot 4</td>
</tr>
<tr>
<td>3 \cdot 2 \cdot 2</td>
</tr>
</tbody>
</table>

Notice that these factors are all prime.

The prime factorization of a number is the number written as the product of its prime factors.
Writing Prime Factorizations

Write the prime factorization of each number.

A. 36

Method 1: Use a factor tree.
Choose any two factors of 36 to begin. Keep finding factors until each branch ends at a prime factor.

```
36
\(\underline{2} \cdot 18\)
\(\underline{9} \cdot 2\)
\(\underline{3} \cdot 3\)
```

36 = \(2 \cdot 3 \cdot 2 \cdot 3\)

The prime factorization of 36 is \(2 \cdot 3 \cdot 2 \cdot 3\), or \(2^2 \cdot 3^2\).

B. 54

Method 2: Use a ladder diagram.
Choose a prime factor of 54 to begin. Keep dividing by prime factors until the quotient is 1.

```
54
\(\underline{2} \cdot 27\)
\(\underline{3} \cdot 9\)
\(\underline{3} \cdot 3\)
```

54 = \(3 \cdot 3 \cdot 2 \cdot 3\)

The prime factorization of 54 is \(3 \cdot 3 \cdot 2 \cdot 3\), or \(3^3 \cdot 2\).

In Example 2, notice that the prime factors may be written in a different order, but they are still the same factors. Except for changes in the order, there is only one way to write the prime factorization of a number.

Think and Discuss

1. **Tell** how you know when you have found all of the factors of a number.

2. **Tell** how you know when you have found the prime factorization of a number.

3. **Explain** the difference between factors of a number and prime factors of a number.
GUIDED PRACTICE

See Example 1
List all of the factors of each number.

1. 12  2. 21  3. 52  4. 75

See Example 2
Write the prime factorization of each number.

5. 48  6. 20  7. 66  8. 34

INDEPENDENT PRACTICE

See Example 1
List all of the factors of each number.

9. 24  10. 37  11. 42  12. 56
13. 67  14. 72  15. 85  16. 92

See Example 2
Write the prime factorization of each number.

17. 49  18. 38  19. 76  20. 60
21. 81  22. 132  23. 140  24. 87

PRACTICE AND PROBLEM SOLVING

Write each number as a product in two different ways.

25. 34  26. 82  27. 88  28. 50
29. 15  30. 78  31. 94  32. 35

33. **Sports**  Little League Baseball began in 1939 in Pennsylvania. When it first started, there were 45 boys on 3 teams.
   a. If the teams were equally sized, how many boys were on each team?
   b. Name another way the boys could have been divided into equally sized teams. (Remember that a baseball team must have at least 9 players.)

34. **Critical Thinking**  Use the divisibility rules to list the factors of 171. Explain how you determined the factors.

Find the prime factorization of each number.

35. 99  36. 249  37. 284  38. 620
39. 840  40. 150  41. 740  42. 402

43. The prime factorization of 50 is $2 \cdot 5^2$. Without dividing or using a diagram, find the prime factorization of 100.

44. **Geometry**  The area of a rectangle is the product of its length and width. Suppose the area of a rectangle is 24 in$^2$. What are the possible whole number measurements of its length and width?

45. **Physical Science**  The speed of sound at sea level at 20°C is 343 meters per second. Write the prime factorization of 343.
Climate changes, habitat destruction, and overhunting can cause animals and plants to die in large numbers. When the entire population of a species begins to die out, the species is considered endangered.

The graph shows the number of endangered species in each category of animal.

46. How many species of mammals are endangered? Write this number as the product of prime factors.

47. Which categories of animals have a prime number of endangered species?

48. How many species of reptiles and amphibians combined are endangered? Write the answer as the product of prime factors.

49. **What’s the Error?** When asked to write the prime factorization of the number of endangered amphibian species, a student wrote $3 \times 9$. Explain the error and write the correct answer.

50. **Write About It** A team of five scientists is going to study endangered insect species. The scientists want to divide the species evenly among them. Will they be able to do this? Why or why not?

51. **Challenge** Add the number of endangered mammal species to the number of endangered bird species. Find the prime factorization of this number.

---

**Life Science**

Laysan albatross chicks often die from eating plastic that pollutes the oceans and beaches. Clean-up efforts may prevent the albatross from becoming endangered.

---

52. **Multiple Choice** Which expression shows the prime factorization of 50?

A. $2 \times 5^2$
B. $2 \times 5^{10}$
C. $10^5$
D. $5 \times 10$

53. **Gridded Response** What number has a prime factorization of $2 \times 2 \times 3 \times 5$?

54. Damien’s favorite song is 4.2 minutes long. Jan’s favorite song is 2.89 minutes long. Estimate the difference in the lengths of the songs by rounding to the nearest whole number. (Lesson 3-2)

Tell whether each number is divisible by 2, 3, 4, 5, 6, 9, and 10. (Lesson 4-1)

55. 105
56. 198
57. 360
58. 235
59. 100
60. 92
61. 540
62. 441
Factors shared by two or more whole numbers are called common factors. The largest of the common factors is called the greatest common factor, or GCF.

Factors of 24: 1, 2, 3, 4, 6, 8, 12, 24
Factors of 36: 1, 2, 3, 4, 6, 9, 12, 18, 36
Common factors: 1, 2, 3, 4, 6, (12)
The greatest common factor (GCF) of 24 and 36 is 12.

Example 1 shows three different methods for finding the GCF.

**Example 1**

**Finding the GCF**

Find the GCF of each set of numbers.

**A** 16 and 24

Method 1: List the factors.

factors of 16: 1, 2, 4, 8, 16  
Factors of 24: 1, 2, 3, 6, 8, 12, 24
The GCF of 16 and 24 is 8.

**B** 12, 24, and 32

Method 2: Use prime factorization.

12 = \(2 \cdot 2 \cdot 3\)
24 = \(2 \cdot 2 \cdot 2 \cdot 3\)
32 = \(2 \cdot 2 \cdot 2 \cdot 2 \cdot 2\)

The GCF of 12, 24, and 32 is 4.

**C** 12, 18, and 60

Method 3: Use a ladder diagram.

Start with 2 and divide into each number. Keep dividing until the three numbers have no common factors.

\[
\begin{array}{cccc}
2 & 12 & 18 & 60 \\
3 & 6 & 9 & 30 \\
2 & 3 & 10 \\
\end{array}
\]

2 \(\times\) 3 = 6
The GCF is 6.
EXAMPLE 2

PROBLEM SOLVING APPLICATION

There are 12 boys and 18 girls in Mr. Ruiz’s science class. The students must form lab groups. Each group must have the same number of boys and the same number of girls. What is the greatest number of groups Mr. Ruiz can make if every student must be in a group?

1. Understand the Problem

The answer will be the greatest number of groups 12 boys and 18 girls can form so that each group has the same number of boys, and each group has the same number of girls.

2. Make a Plan

You can make an organized list of the possible groups.

3. Solve

There are more girls than boys in the class, so there will be more girls than boys in each group.

<table>
<thead>
<tr>
<th>Boys</th>
<th>Girls</th>
<th>Groups</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>[B GG, B GG, B GG] 9 boys, 18 girls: There are 3 boys not in groups. ✗</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>[BB GGG, BB GGG, BB GGG] 12 boys, 18 girls: Every student is in a group. ✓</td>
</tr>
</tbody>
</table>

The greatest number of groups is 6.

4. Look Back

The number of groups will be a common factor of the number of boys and the number of girls. To form the largest number of groups, find the GCF of 12 and 18.

Factors of 12: 1, 2, 3, 4, 6, 12
Factors of 18: 1, 2, 3, 6, 9, 18

The GCF of 12 and 18 is 6.

Think and Discuss

1. Explain what the GCF of two prime numbers is.
2. Tell what the least common factor of a group of numbers would be.
GUIDED PRACTICE

Find the GCF of each set of numbers.

1. 18 and 27  \hspace{1cm} 2. 32 and 72  \hspace{1cm} 3. 21, 42, and 56
4. 15, 30, and 60  \hspace{1cm} 5. 18, 24, and 36  \hspace{1cm} 6. 9, 36, and 81

7. Kim is making flower arrangements. She has 16 red roses and 20 pink roses. Each arrangement must have the same number of red roses and the same number of pink roses. What is the greatest number of arrangements Kim can make if every flower is used?

INDEPENDENT PRACTICE

Find the GCF of each set of numbers.

8. 10 and 35  \hspace{1cm} 9. 28 and 70  \hspace{1cm} 10. 36 and 72
11. 26, 48, and 62  \hspace{1cm} 12. 16, 40, and 88  \hspace{13.81cm} 13. 12, 60, and 68
14. 30, 45, and 75  \hspace{1cm} 15. 24, 48, and 84  \hspace{1cm} 16. 16, 48, and 72

17. The local recreation center held a scavenger hunt. There were 15 boys and 9 girls at the event. The group was divided into the greatest number of teams possible with the same number of boys on each team and the same number of girls on each team. How many teams were made if each person was on a team?

18. Ms. Kline makes balloon arrangements. She has 32 blue balloons, 24 yellow balloons, and 16 white balloons. Each arrangement must have the same number of each color. What is the greatest number of arrangements that Ms. Kline can make if every balloon is used?

PRACTICE AND PROBLEM SOLVING

Write the GCF of each set of numbers.

19. 60 and 84  \hspace{1cm} 20. 14 and 17  \hspace{1cm} 21. 10, 35, and 110
22. 21 and 306  \hspace{1cm} 23. 630 and 712  \hspace{1cm} 24. 16, 24, and 40
25. 75, 225, and 150  \hspace{1cm} 26. 42, 112, and 105  \hspace{1cm} 27. 12, 16, 20, and 24

28. Jared has 12 jars of grape jam, 16 jars of strawberry jam, and 24 jars of raspberry jam. He wants to place the jam into the greatest possible number of boxes so that each box has the same number of jars of each kind of jam. How many boxes does he need?

29. Pam is making fruit baskets. She has 30 apples, 24 bananas, and 12 oranges. What is the greatest number of baskets she can make if each type of fruit is distributed equally among the baskets?

30. Critical Thinking Write a set of three different numbers that have a GCF of 9. Explain your method.
Write the GCF of each set of numbers.

31. 16, 24, 30, and 42  
32. 25, 90, 45, and 100  
33. 27, 90, 135, and 72  
34. \(2 \times 2 \times 3\) and \(2 \times 2\)  
35. \(2 \times 3^2 \times 7\) and \(2^2 \times 3\)  
36. \(3^2 \times 7\) and \(2 \times 3 \times 5^2\)

37. Mr. Chu is planting 4 types of flowers in his garden. He wants each row to contain the same number of each type of flower. What is the greatest number of rows Mr. Chu can plant if every bulb is used?

38. In a parade, one school band will march directly behind another school band. All rows must have the same number of students. The first band has 36 students, and the second band has 60 students. What is the greatest number of students who can be in each row?

39. **Social Studies** Branches of the U.S. Mint in Denver and Philadelphia make all U.S. coins for circulation. A tiny \(D\) or \(P\) on the coin tells you where the coin was minted. Suppose you have 32 \(D\) quarters and 36 \(P\) quarters. What is the greatest number of groups you can make with the same number of \(D\) quarters in each group and the same number of \(P\) quarters in each group so that every quarter is placed in a group?

40. **What’s the Error?** Mike says if \(12 = 2^2 \cdot 3\) and \(24 = 2^3 \cdot 3\), then the GCF of 12 and 24 is \(2 \cdot 3\), or 6. Explain Mike’s error.

41. **Write About It** What method do you like best for finding the GCF? Why?

42. **Challenge** The GCF of three numbers is 9. The sum of the numbers is 90. Find the three numbers.

### Test Prep and Spiral Review

43. **Multiple Choice** For which set of numbers is 16 the GCF?
   - A. 16, 32, 48
   - B. 12, 24, 32
   - C. 24, 48, 60
   - D. 8, 80, 100

44. **Multiple Choice** Mrs. Lyndon is making baskets of muffins. She has 48 lemon muffins, 120 blueberry muffins, and 112 banana nut muffins. How many baskets can Mrs. Lyndon make with each type of muffin distributed evenly?
   - F. 4
   - G. 6
   - H. 8
   - J. 12

Solve each equation. *(Lessons 2-4, 2-5, 2-6, 2-7)*

45. \(y + 37 = 64\)
46. \(c - 5 = 19\)
47. \(72 \div z = 9\)
48. \(3v = 81\)

Write the prime factorization of each number. *(Lesson 4-2)*

49. 42
50. 19
51. 51
52. 132
53. 200
Greatest Common Factor

You can use a graphing calculator to quickly find the greatest common factor (GCF) of two or more numbers. A calculator is particularly useful when you need to find the GCF of large numbers.

**Activity**

Find the GCF of 504 and 3,150.

The GCF is also known as the **greatest common divisor**, or GCD. The GCD function is found on the MATH menu.

To find the GCD on a graphing calculator, press \( \boxed{\text{MATH}} \). Press \( \boxed{\downarrow} \) to highlight \( \boxed{\text{NUM}} \), and then use \( \boxed{\downarrow} \) to scroll down and highlight \( \boxed{9} \).

Press \( \boxed{\text{ENTER}} \), \( \boxed{504} \), \( \boxed{3150} \), \( \boxed{\text{ENTER}} \).

The greatest common factor of 504 and 3,150 is 126.

**Think and Discuss**

1. Suppose your calculator will not allow you to enter three numbers into the GCD function. How could you still use your calculator to find the GCF of the three following numbers: 4,896; 2,364; and 656? Explain your strategy and why it works.

2. Would you use your calculator to find the GCF of 6 and 18? Why or why not?

**Try This**

Find the GCF of each set of numbers.

1. 14, 48  
2. 18, 54  
3. 99, 121  
4. 144, 196  
5. 200, 136  
6. 246, 137  
7. 72, 860  
8. 55, 141, 91
Quiz for Lessons 4-1 Through 4-3

4-1 Divisibility

Tell whether each number is divisible by 2, 3, 4, 5, 6, 9, and 10.

1. 708
2. 514
3. 470
4. 338
5. A highway loop around a city is 45 miles long. If exits are placed every 5 miles, will the exits be evenly spaced around the loop? Explain.
6. Hoover Dam is 1,244 feet across at the top. Tell whether this number is divisible by 2, 3, 4, 5, 6, 9, and 10.

Tell whether each number is prime or composite.

7. 76
8. 59
9. 69
10. 33

4-2 Factors and Prime Factorization

List all of the factors of each number.

11. 26
12. 32
13. 39
14. 84
15. Mr. Collins’s bowling league has 48 members. If the league splits into teams of 12 members each, how many equally sized teams will there be?

Write the prime factorization of each number.

16. 96
17. 50
18. 104
19. 63
20. Scientists classify many sunflowers in the genus *Helianthus*. There are approximately 67 species of *Helianthus*. Write the prime factorization of 67.

4-3 Greatest Common Factor

Find the GCF of each set of numbers.

21. 16 and 36
22. 22 and 88
23. 65 and 91
24. 20, 55, and 85
25. There are 36 sixth-graders and 40 seventh-graders. What is the greatest number of teams that the students can form if each team has the same number of sixth-graders and the same number of seventh-graders and every student must be on a team?
26. There are 14 girls and 21 boys in Mrs. Sutter’s gym class. To play a certain game, the students must form teams. Each team must have the same number of girls and the same number of boys. What is the greatest number of teams Mrs. Sutter can make if every student is on a team?
27. Mrs. Young, an art teacher, is organizing the art supplies. She has 76 red markers, 52 blue markers, and 80 black markers. She wants to divide the markers into boxes with the same number of red, the same number of blue, and the same number of black markers in each box. What is the greatest number of boxes she can have if every marker is placed in a box?
Understand the Problem

- Interpret unfamiliar words

You must understand the words in a problem in order to solve it. If there is a word you do not know, try to use context clues to figure out its meaning. Suppose there is a problem about red, green, blue, and chartreuse fabric. You may not know the word *chartreuse*, but you can guess that it is probably a color. To make the problem easier to understand, you could replace *chartreuse* with the name of a familiar color, like *white*.

In some problems, the name of a person, place, or thing might be difficult to pronounce, such as *Mr. Joubert*. When you see a proper noun that you do not know how to pronounce, you can use another proper noun or a pronoun in its place. You could replace *Mr. Joubert* with *he*. You could replace *Koenisburg Street* with *K Street*.

Grace is making flower bouquets. She has 18 chrysanthemums and 42 roses. She wants to arrange them in groups that each have the same number of chrysanthemums and the same number of roses. What is the fewest number of flowers that Grace can have in each group? How many chrysanthemums and how many roses will be in each group?

Most marbles are made from glass. The glass is liquefied in a furnace and poured. It is then cut into cylinders that are rounded off and cooled. Suppose 1,200 cooled marbles are put into packages of 8. How many packages could be made? Would there be any marbles left over?

In ancient times, many civilizations used calendars that divided the year into months of 30 days. A year has 365 days. How many whole months were in these ancient calendars? Were there any days left over? If so, how many?

Mrs. LeFeubre is tiling her garden walkway. It is a rectangle that is 4 feet wide and 20 feet long. Mrs. LeFeubre wants to use square tiles, and she does not want to have to cut any tiles. What is the size of the largest square tile that Mrs. LeFeubre can use?
Explore Decimals and Fractions

Use with Lesson 4-4

You can use decimal grids to show the relationship between fractions and decimals.

Activity

Write the number represented on each grid as a fraction and as a decimal.

1. Seven hundredths squares are shaded → 0.07
   How many squares are shaded? 7
   How many squares are in the whole? 100
   \[ \frac{7}{100} \]
   \[ 0.07 = \frac{7}{100} \]

2. Three tenths columns are shaded → 0.3
   How many complete columns are shaded? 3
   How many columns are in the whole? 10
   \[ \frac{3}{10} \]
   \[ 0.3 = \frac{3}{10} \]

Think and Discuss

1. Is 0.09 the same as \( \frac{9}{100} \)? Use decimal grids to support your answer.

Try This

Use decimal grids to represent each number.

1. 0.8
2. \( \frac{37}{100} \)
3. 0.53
4. \( \frac{1}{10} \)
5. \( \frac{67}{100} \)

6. For 1–5, write each decimal as a fraction and each fraction as a decimal.
Decimals and fractions can often be used to represent the same number. For example, a baseball player’s or baseball team’s batting average can be represented as a fraction:

\[
\frac{\text{number of hits}}{\text{number of times at bat}}
\]

In 2005, the University of Texas baseball team won its sixth College World Series title. During that season, the team had 734 hits and 2,432 at bats. The team’s batting average was \(\frac{734}{2,432}\) .

\[
734 \div 2,432 = 0.3018092105…
\]

The 2005 batting average for the University of Texas baseball team is reported as .302.

Decimals can be written as fractions or mixed numbers. A number that contains both a whole number greater than 0 and a fraction, such as \(1\frac{3}{4}\), is called a **mixed number**.

<table>
<thead>
<tr>
<th>Place Value</th>
<th>Mixed numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ones</td>
<td>0</td>
</tr>
<tr>
<td>Tenths</td>
<td>0.25</td>
</tr>
<tr>
<td>Hundredths</td>
<td>0.5</td>
</tr>
<tr>
<td>Thousandths</td>
<td>0.75</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>(\frac{1}{4})</td>
<td>1.25</td>
</tr>
<tr>
<td>(\frac{1}{2})</td>
<td>1.5</td>
</tr>
<tr>
<td>(\frac{3}{4})</td>
<td>1.75</td>
</tr>
<tr>
<td>(\frac{1}{4})</td>
<td>2</td>
</tr>
<tr>
<td>(\frac{1}{2})</td>
<td>2.25</td>
</tr>
<tr>
<td>(\frac{3}{4})</td>
<td>2.5</td>
</tr>
</tbody>
</table>

**EXAMPLE 1**

**Writing Decimals as Fractions or Mixed Numbers**

Write each decimal as a fraction or mixed number.

**A**

0.23

Identify the place value of the digit farthest to the right.

\[
\frac{23}{100}
\]

The 3 is in the hundredths place, so use 100 as the denominator.

**B**

1.7

Identify the place value of the digit farthest to the right.

\[
1\frac{7}{10}
\]

Write the whole number, 1. The 7 is in the tenths place, so use 10 as the denominator.
**Writing Fractions as Decimals**

Write each fraction or mixed number as a decimal.

**Example 2**

A \[ \frac{3}{4} \]

- Divide 3 by 4.
- Add zeros after the decimal point.
- The remainder is 0.

\[ \frac{3}{4} = 0.75 \]

B \[ \frac{5}{3} \]

- Divide 2 by 3.
- Add zeros after the decimal point.
- The 6 repeats in the quotient.

\[ \frac{5}{3} = 5.666... = 5.\overline{6} \]

A **terminating decimal**, such as 0.75, has a finite number of decimal places. A **repeating decimal**, such as 0.666..., has a block of one or more digits that repeat continuously.

<table>
<thead>
<tr>
<th>Common Fractions and Equivalent Decimals</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{5} )</td>
</tr>
<tr>
<td>0.2</td>
</tr>
</tbody>
</table>

**Example 3**

Comparing and Ordering Fractions and Decimals

Order the fractions and decimals from least to greatest.

0.5, \( \frac{1}{5} \), 0.37

First rewrite the fraction as a decimal. \( \frac{1}{5} = 0.2 \)

Order the three decimals.

The numbers in order from least to greatest are \( \frac{1}{5} \), 0.37, and 0.5.

**Think and Discuss**

1. **Tell** how reading the decimal 6.9 as “six and nine tenths” helps you to write 6.9 as a mixed number.

2. **Look** at the decimal 0.121122111222... If the pattern continues, is this a repeating decimal? Why or why not?
4-4 Exercises

**GUIDED PRACTICE**

See Example 1

Write each decimal as a fraction or mixed number.

1. 0.15  
2. 1.25  
3. 0.43  
4. 2.6

See Example 2

Write each fraction or mixed number as a decimal.

5. \(\frac{2}{5}\)  
6. \(2\frac{7}{8}\)  
7. \(\frac{1}{8}\)  
8. \(4\frac{1}{10}\)

See Example 3

Order the fractions and decimals from least to greatest.

9. \(\frac{2}{3}\), 0.78, 0.21  
10. \(\frac{5}{16}\), 0.67, \(\frac{1}{6}\)  
11. 0.52, \(\frac{1}{9}\), 0.3

**INDEPENDENT PRACTICE**

See Example 1

Write each decimal as a fraction or mixed number.

12. 0.31  
13. 5.71  
14. 0.13  
15. 3.23  
16. 0.5  
17. 2.7  
18. 0.19  
19. 6.3

See Example 2

Write each fraction or mixed number as a decimal.

20. \(\frac{1}{9}\)  
21. \(1\frac{3}{5}\)  
22. \(\frac{8}{9}\)  
23. \(3\frac{11}{40}\)  
24. \(2\frac{5}{6}\)  
25. \(\frac{3}{8}\)  
26. \(4\frac{4}{5}\)  
27. \(\frac{5}{8}\)

See Example 3

Order the fractions and decimals from least to greatest.

28. 0.49, 0.82, \(\frac{1}{2}\)  
29. \(\frac{3}{8}\), 0.29, \(\frac{1}{9}\)  
30. 0.94, \(\frac{4}{5}\), 0.6  
31. 0.11, \(\frac{1}{10}\), 0.13  
32. \(\frac{2}{3}\), 0.42, \(\frac{2}{5}\)  
33. \(\frac{3}{7}\), 0.76, 0.31

**PRACTICE AND PROBLEM SOLVING**

Write each decimal in expanded form and use a whole number or fraction for each place value.

34. 0.81  
35. 92.3  
36. 13.29  
37. 107.17

Write each fraction as a decimal. Tell whether the decimal terminates or repeats.

38. \(\frac{7}{9}\)  
39. \(\frac{1}{6}\)  
40. \(\frac{17}{20}\)  
41. \(\frac{5}{12}\)  
42. \(\frac{7}{8}\)  
43. \(\frac{4}{5}\)  
44. \(\frac{9}{5}\)  
45. \(\frac{15}{18}\)  
46. \(\frac{7}{3}\)  
47. \(\frac{11}{12}\)

Compare. Write <, >, or =.

48. 0.75 \[\frac{3}{4}\]  
49. \(\frac{5}{8}\) \[0.5\]  
50. 0.78 \[\frac{7}{9}\]  
51. \(\frac{1}{3}\) \[0.35\]  
52. \(\frac{2}{5}\) \[0.4\]  
53. 0.75 \[\frac{4}{5}\]  
54. \(\frac{3}{8}\) \[0.25\]  
55. 0.8 \[\frac{5}{6}\]  
56. **Multi-Step** Peter walked \(1\frac{3}{5}\) miles on a treadmill. Sally walked 1.5 miles on the treadmill. Who walked farther? Explain.
Order the mixed numbers and decimals from greatest to least.

57. 4.48, 3.92, 4\(\frac{1}{2}\) 58. 10\(\frac{5}{9}\), 10.5, 10\(\frac{1}{5}\) 59. 125.205, 125.25, 125\(\frac{1}{5}\)

**Sports** The table shows batting averages for two baseball seasons. Use the table for Exercises 60–62.

<table>
<thead>
<tr>
<th>Player</th>
<th>Season 1</th>
<th>Season 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pedro</td>
<td>0.360</td>
<td>(\frac{3}{10})</td>
</tr>
<tr>
<td>Jill</td>
<td>0.380</td>
<td>(\frac{3}{8})</td>
</tr>
<tr>
<td>Lamar</td>
<td>0.290</td>
<td>(\frac{1}{3})</td>
</tr>
<tr>
<td>Britney</td>
<td>0.190</td>
<td>(\frac{3}{20})</td>
</tr>
</tbody>
</table>

60. Which players had higher batting averages in season 1 than they had in season 2?

61. Who had the highest batting average in either season?

62. **Multi-Step** Whose batting average changed the most between season 1 and season 2?

63. **Life Science** Most people with color deficiency (often called color blindness) have trouble distinguishing shades of red and green. About 0.05 of men in the world have color deficiency. What fraction of men have color deficiency?

64. **What’s the Error?** A student found the decimal equivalent of \(\frac{7}{18}\) to be 0.3\(\frac{3}{8}\). Explain the error. What is the correct answer?

65. **Write About It** The decimal for \(\frac{1}{25}\) is 0.04, and the decimal for \(\frac{2}{25}\) is 0.08. Without dividing, find the decimal for \(\frac{6}{25}\). Explain how you found your answer.

66. **Challenge** Write \(\frac{1}{999}\) as a decimal.

---

**Test Prep and Spiral Review**

67. **Multiple Choice** Which numbers are listed from least to greatest?

- A 0.65, 0.81, \(\frac{4}{5}\)
- B 0.81, 0.65, \(\frac{4}{5}\)
- C \(\frac{4}{5}\), 0.81, 0.65
- D 0.65, \(\frac{4}{5}\), 0.81

68. **Gridded Response** Write \(\frac{5\frac{1}{8}}{8}\) as a decimal.

Find each sum or difference. (Lesson 3-3)

69. 12.56 + 8.91  70. 19.05 − 2.27  71. 5 + 8.25 + 10.2

Find the GCF of each set of numbers. (Lesson 4-3)

72. 235 and 35  73. 28 and 154  74. 90 and 56  75. 16 and 112

---

184  *Chapter 4 Number Theory and Fractions*
Pattern blocks can be used to model equivalent fractions. To find a fraction that is equivalent to $\frac{1}{2}$, first choose the pattern block that represents $\frac{1}{2}$. Then find all the pieces of one color that will fit evenly on the $\frac{1}{2}$ block. Count these pieces to find the equivalent fraction. You may be able to find more than one equivalent fraction.

Activity

Use pattern blocks to find an equivalent fraction for $\frac{8}{12}$.

Think and Discuss

1. Can you find a combination of pattern blocks for $\frac{1}{3}$? Find an equivalent fraction for $\frac{1}{3}$.
2. Are $\frac{9}{12}$ and $\frac{3}{6}$ equivalent? Use pattern blocks to support your answer.

Try This

Write the fraction that is modeled. Then find an equivalent fraction.

1. 
2. 

4-5A Hands-On Lab 185
Learn to write equivalent fractions.

Vocabulary
- equivalent fractions
- simplest form

Rulers often have marks for inches, \( \frac{1}{2} \), \( \frac{1}{4} \), and \( \frac{1}{8} \) inches.

Notice that \( \frac{1}{2} \) in., \( \frac{2}{4} \) in., and \( \frac{4}{8} \) in. all name the same length. Fractions that represent the same value are equivalent fractions. So \( \frac{1}{2}, \frac{2}{4}, \) and \( \frac{4}{8} \) are equivalent fractions.

### Example 1
Finding Equivalent Fractions

Find two equivalent fractions for \( \frac{6}{8} \).

The same area is shaded when the rectangle is divided into 8 parts, 12 parts, and 4 parts.

So \( \frac{6}{8}, \frac{9}{12}, \) and \( \frac{3}{4} \) are all equivalent fractions.

### Example 2
Multiplying and Dividing to Find Equivalent Fractions

Find the missing number that makes the fractions equivalent.

\[
\frac{2}{3} = \frac{?}{18}
\]

\[
\frac{2 \cdot 6}{3 \cdot 6} = \frac{12}{18}
\]

In the denominator, 3 is multiplied by 6 to get 18. Multiply the numerator, 2, by the same number, 6.

So \( \frac{2}{3} \) is equivalent to \( \frac{12}{18} \).
Every fraction has one equivalent fraction that is called the simplest form of the fraction. A fraction is in **simplest form** when the GCF of the numerator and the denominator is 1.

Example 3 shows two methods for writing a fraction in simplest form.

### Writing Fractions in Simplest Form

**Write each fraction in simplest form.**

**A.** $\frac{18}{24}$

The GCF of 18 and 24 is 6, so $\frac{18}{24}$ is not in simplest form.

**Method 1: Use the GCF.**

$\frac{18 \div 6}{24 \div 6} = \frac{3}{4}$  
*Divide 18 and 24 by their GCF, 6.*

**Method 2: Use prime factorization.**

$\frac{18}{24} = \frac{2 \cdot 3 \cdot 3}{2 \cdot 2 \cdot 3 \cdot 3} = \frac{3}{4}$  
*Write the prime factors of 18 and 24. Simplify.*

So $\frac{18}{24}$ written in simplest form is $\frac{3}{4}$.

**B.** $\frac{8}{9}$

The GCF of 8 and 9 is 1, so $\frac{8}{9}$ is already in simplest form.

---

**Think and Discuss**

1. **Explain** whether a fraction is equivalent to itself.

2. **Tell** which of the following fractions are in simplest form: $\frac{9}{21}$, $\frac{20}{25}$, and $\frac{5}{13}$. Explain.

3. **Explain** how you know that $\frac{7}{16}$ is in simplest form.
## 4-5 Exercises

### Guided Practice

**See Example 1**

Find two equivalent fractions for each fraction.

1. \(\frac{4}{6}\)  
2. \(\frac{3}{12}\)  
3. \(\frac{3}{6}\)  
4. \(\frac{6}{16}\)

### Find the missing numbers that make the fractions equivalent.

5. \(\frac{2}{5} = \frac{10}{\_}\)  
6. \(\frac{7}{21} = \frac{1}{\_}\)  
7. \(\frac{3}{4} = \frac{\_}{28}\)  
8. \(\frac{8}{12} = \frac{\_}{3}\)

### Write each fraction in simplest form.

9. \(\frac{2}{10}\)  
10. \(\frac{6}{18}\)  
11. \(\frac{4}{16}\)  
12. \(\frac{9}{15}\)

### Independent Practice

**See Example 1**

Find two equivalent fractions for each fraction.

13. \(\frac{3}{9}\)  
14. \(\frac{2}{10}\)  
15. \(\frac{3}{21}\)  
16. \(\frac{3}{18}\)  
17. \(\frac{12}{15}\)  
18. \(\frac{4}{10}\)  
19. \(\frac{10}{12}\)  
20. \(\frac{6}{10}\)

### Find the missing numbers that make the fractions equivalent.

21. \(\frac{3}{7} = \frac{\_}{35}\)  
22. \(\frac{6}{48} = \frac{1}{\_}\)  
23. \(\frac{2}{5} = \frac{28}{\_}\)  
24. \(\frac{12}{18} = \frac{\_}{3}\)  
25. \(\frac{2}{7} = \frac{\_}{21}\)  
26. \(\frac{8}{32} = \frac{1}{\_}\)  
27. \(\frac{2}{7} = \frac{40}{\_}\)  
28. \(\frac{3}{5} = \frac{21}{\_}\)

### Write each fraction in simplest form.

29. \(\frac{2}{8}\)  
30. \(\frac{10}{15}\)  
31. \(\frac{6}{30}\)  
32. \(\frac{6}{14}\)  
33. \(\frac{12}{16}\)  
34. \(\frac{4}{28}\)  
35. \(\frac{4}{8}\)  
36. \(\frac{10}{35}\)

### Practice and Problem Solving

**Extra Practice**

Write the equivalent fractions represented by each picture.

37.  
38.  
39.  
40.  

Write each fraction in simplest form. Show two ways to simplify.

41. \(\frac{5}{20}\)  
42. \(\frac{4}{52}\)  
43. \(\frac{14}{35}\)  
44. \(\frac{112}{220}\)
45. You can buy food, such as southern sesame seed cookies, at \( \frac{1}{10} \) of the booths. Write two equivalent fractions for \( \frac{1}{10} \).

46. Handwoven sweetgrass baskets are a regional specialty. About 8 out of every 10 baskets sold are woven at the market. Write a fraction for “8 out of 10.” Then write this fraction in simplest form.

47. Suppose the circle graph shows the number of each kind of craft booth at the Old City Market. For each type of booth, tell what fraction it represents of the total number of craft booths. Write these fractions in simplest form.

48. Customers can buy packages of dried rice and black-eyed peas, which can be made into black-eyed pea soup. One recipe for black-eyed pea soup calls for \( \frac{1}{2} \) tsp of basil. How could you measure the basil if you had only a \( \frac{1}{4} \) tsp measuring spoon? What if you had only a \( \frac{1}{8} \) tsp measuring spoon?

49. Write About It The recipe for soup also calls for \( \frac{1}{4} \) tsp of pepper. How many fractions are equivalent to \( \frac{1}{4} \)? Explain.

50. Challenge Silver jewelry is a popular item at the market. Suppose there are 28 bracelets at one jeweler’s booth and that \( \frac{3}{7} \) of these bracelets have red stones. How many bracelets have red stones?
Look at a standard ruler. It probably has marks for inches, half inches, quarter inches, eighth inches, and sixteenth inches.

In this activity, you will make some of your own rulers and use them to help you find and understand equivalent fractions.

**Activity**

1. You will need four strips of paper. On one strip, use your ruler to make a mark for every half inch. Number each mark, beginning with 0. Label this strip “half-inch ruler.”

On a second strip, make a mark for every quarter inch. Again, number each mark, beginning with 0. Label this strip “quarter-inch ruler.”

Do the same thing for eighth inches and sixteenth inches.
2. Now use the half-inch ruler you made to measure the green line segment at right. How many half inches long is the segment?

Use your quarter-inch ruler to measure the line segment again. How many quarter inches long is the segment?

How many eighth inches long is the segment?

How many sixteenth inches?

Fill in the blanks: \( \frac{1}{2} = \frac{4}{8} = \frac{8}{16} \).

3. Use your quarter-inch ruler to measure the green line segment below.

How long is the segment?

Now use your eighth-inch ruler to measure the line segment again. How many eighth inches long is the segment?

How many sixteenth inches?

Fill in the blanks: \( \frac{3}{4} = \frac{6}{8} = \frac{12}{16} \).

**Think and Discuss**

1. How does a ruler show that equivalent fractions have the same value?

2. Look at your lists of equivalent fractions from 2 and 3. Do you notice any patterns? Describe them.

3. Use your rulers to measure an object longer than 1 inch. Use your measurements to write equivalent fractions. What do you notice about these fractions?

**Try This**

1. Use your rulers to measure the length of the items below. Use your measurements to write equivalent fractions.

2. Use your rulers to measure several items in your classroom. Use your measurements to write equivalent fractions.
Mixed Numbers and Improper Fractions

Have you ever witnessed a total eclipse of the sun? It occurs when the sun’s light is completely blocked out. A total eclipse is rare—only three have been visible in the continental United States since 1963.

The graph shows that the eclipse in 2017 will last $2\frac{3}{4}$ minutes. There are eleven $\frac{1}{4}$-minute sections, so $2\frac{3}{4} = \frac{11}{4}$.

An improper fraction is a fraction in which the numerator is greater than or equal to the denominator, such as $\frac{11}{4}$.

Whole numbers can be written as improper fractions. The whole number is the numerator, and the denominator is 1. For example, $7 = \frac{7}{1}$.

When the numerator is less than the denominator, the fraction is called a proper fraction.

<table>
<thead>
<tr>
<th>Improper and Proper Fractions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Improper Fractions</strong></td>
</tr>
<tr>
<td>• Numerator equals denominator ➔ fraction is equal to 1</td>
</tr>
<tr>
<td>$\frac{3}{3} = 1$  $\frac{102}{102} = 1$</td>
</tr>
<tr>
<td>• Numerator greater than denominator ➔ fraction is greater than 1</td>
</tr>
<tr>
<td>$\frac{9}{5} &gt; 1$  $\frac{13}{115} &gt; 1$</td>
</tr>
<tr>
<td><strong>Proper Fractions</strong></td>
</tr>
<tr>
<td>• Numerator less than denominator ➔ fraction is less than 1</td>
</tr>
<tr>
<td>$\frac{2}{5} &lt; 1$  $\frac{102}{351} &lt; 1$</td>
</tr>
</tbody>
</table>

You can write an improper fraction as a mixed number.

**Example 1**

Astronomy Application

The longest total solar eclipse in the next 200 years will take place in 2186. It will last about $\frac{15}{2}$ minutes. Write $\frac{15}{2}$ as a mixed number.

Method 1: Use a model.

Draw squares divided into half sections. Shade 15 of the half sections.

There are 7 whole squares and 1 half square, or $7\frac{1}{2}$ squares, shaded.
Method 2: Use division.

\[
\begin{array}{c}
2 \overline{) 15} \\
2) 15 \\
- 14 \\
\hline
1
\end{array}
\]

Divide the numerator by the denominator.

To form the fraction part of the quotient, use the remainder as the numerator and the divisor as the denominator.

The 2186 eclipse will last about \(7\frac{1}{2}\) minutes.

Mixed numbers can be written as improper fractions.

**Example 2**

**Writing Mixed Numbers as Improper Fractions**

Write \(2\frac{1}{5}\) as an improper fraction.

Method 1: Use a model.

You can draw a diagram to illustrate the whole and fractional parts.

\[
\begin{array}{c}
\text{There are 11 fifths, or } \frac{11}{5}. \\
\text{Count the fifths in the diagram.}
\end{array}
\]

Method 2: Use multiplication and addition.

When you are changing a mixed number to an improper fraction, spiral clockwise as shown in the picture. The order of operations will help you remember to multiply before you add.

\[
2\frac{1}{5} = \frac{(5 \cdot 2) + 1}{5}
\]

Multiply the whole number by the denominator and add the numerator. Keep the same denominator.

\[
= \frac{10 + 1}{5}
\]

\[
= \frac{11}{5}
\]

**Think and Discuss**

1. **Read** each improper fraction: \(\frac{10}{7}, \frac{25}{9}, \frac{31}{16}\).

2. **Tell** whether each fraction is less than 1, equal to 1, or greater than 1: \(\frac{21}{21}, \frac{54}{103}, \frac{9}{11}, \frac{7}{3}\).

3. **Explain** why any mixed number written as a fraction will be improper.
1. The fifth largest meteorite found in the United States is named the Navajo. The Navajo weighs $\frac{12}{5}$ tons. Write $\frac{12}{5}$ as a mixed number.

2. Write each mixed number as an improper fraction.
   2. $1\frac{1}{4}$  3. $2\frac{2}{3}$  4. $1\frac{2}{7}$  5. $2\frac{2}{5}$

6. Astronomy Saturn is the sixth planet from the Sun. It takes Saturn $\frac{59}{2}$ years to revolve around the Sun. Write $\frac{59}{2}$ as a mixed number.

7. Astronomy Pluto has low surface gravity. A person who weighs 143 pounds on Earth weighs $\frac{43}{5}$ pounds on Pluto. Write $\frac{43}{5}$ as a mixed number.

Write each mixed number as an improper fraction.
   8. $1\frac{3}{5}$  9. $2\frac{2}{9}$  10. $3\frac{1}{7}$  11. $4\frac{1}{3}$
   12. $2\frac{3}{8}$  13. $4\frac{1}{6}$  14. $1\frac{4}{9}$  15. $3\frac{4}{5}$

Write each improper fraction as a mixed number or whole number. Tell whether your answer is a mixed number or whole number.
   16. $\frac{21}{4}$  17. $\frac{32}{8}$  18. $\frac{20}{3}$  19. $\frac{43}{5}$
   20. $\frac{108}{9}$  21. $\frac{87}{10}$  22. $\frac{98}{11}$  23. $\frac{105}{7}$

Write each mixed number as an improper fraction.
   24. $9\frac{1}{4}$  25. $4\frac{9}{11}$  26. $11\frac{4}{9}$  27. $16\frac{3}{5}$

28. Measurement The actual dimensions of a piece of lumber called a 2-by-4 are $1\frac{1}{2}$ inches and $3\frac{1}{2}$ inches. Write these numbers as improper fractions.

Replace each shape with a number that will make the equation correct.
   29. $\frac{2}{5} = \frac{17}{\text{●}}$  30. $\frac{6}{11} = \frac{83}{\text{●}}$  31. $\frac{1}{9} = \frac{118}{\text{●}}$
   32. $\frac{6}{7} = \frac{55}{\text{●}}$  33. $\frac{9}{10} = \frac{29}{\text{●}}$  34. $\frac{1}{3} = \frac{55}{\text{●}}$

35. Daniel is a costume designer for movies and music videos. He recently purchased $\frac{256}{9}$ yards of metallic fabric for space-suit costumes. Write a mixed number to represent the number of yards of fabric Daniel purchased.

Write the improper fraction as a decimal. Then use $<,$ $>$, or $=$ to compare.
   36. $\frac{7}{5} \square 1.8$  37. $6.875 \square \frac{55}{8}$  38. $\frac{27}{2} \square 13$  39. $\frac{20}{5} \square 4.25$
**Life Science** The table lists the lengths of the longest bones in the human body. Use the table for Exercises 40–42.

40. Write the length of the ulna as an improper fraction. Then do the same for the length of the humerus.

41. Write the length of the fibula as a mixed number. Then do the same for the length of the femur.

42. Use the mixed-number form of each length. Compare the whole-number part of each length to write the bones in order from longest to shortest.

**Social Studies** The European country of Monaco, with an area of only $1 \frac{4}{5} \text{ km}^2$, is one of the smallest countries in the world. Write $1 \frac{4}{5}$ as an improper fraction.

44. **What’s the Question?** The lengths of Victor’s three favorite movies are $1 \frac{11}{4}$ hours, $9 \frac{9}{4}$ hours, and $7 \frac{7}{4}$ hours. The answer is $2 \frac{1}{4}$ hours. What is the question?

45. **Write About It** Draw models representing $\frac{4}{4}$, $\frac{5}{5}$, and $\frac{9}{9}$. Use your models to explain why a fraction whose numerator is the same as its denominator is equal to 1.

46. **Challenge** Write $\frac{65}{12}$ as a decimal.

<table>
<thead>
<tr>
<th>Longest Human Bones</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fibula (outer lower leg)</td>
</tr>
<tr>
<td>Ulna (inner lower arm)</td>
</tr>
<tr>
<td>Femur (upper leg)</td>
</tr>
<tr>
<td>Humerus (upper arm)</td>
</tr>
<tr>
<td>Tibia (inner lower leg)</td>
</tr>
</tbody>
</table>

**Test Prep and Spiral Review**

47. **Multiple Choice** What is $3 \frac{2}{11}$ written as an improper fraction?
   
   - $\text{A} \quad 3 \frac{5}{11}$
   - $\text{B} \quad 3 \frac{5}{3}$
   - $\text{C} \quad 3 \frac{22}{3}$
   - $\text{D} \quad 7 \frac{10}{11}$

48. **Multiple Choice** It takes $\frac{24}{5}$ new pencils placed end to end to be the same length as one yardstick. What is this improper fraction written as a mixed number?

   - $\text{F} \quad 3 \frac{4}{5}$
   - $\text{G} \quad 4 \frac{1}{4}$
   - $\text{H} \quad 4 \frac{1}{5}$
   - $\text{I} \quad 4 \frac{4}{5}$

Order the numbers from least to greatest. (Lesson 1-1)

49. 1,497; 2,560; 1,038
50. 10,462; 9,198; 11,320
51. 4,706; 11,765; 1,765

Estimate a range for each sum. (Lesson 3-2)

52. 19.85 + 6.7 + 12.4
53. 2.456 + 8.3 + 11.05
54. 15.36 + 10.75 + 6.1

List all the factors of each number. (Lesson 4-2)

55. 57
56. 36
57. 54
Quiz for Lessons 4-4 Through 4-6

4-4 Decimals and Fractions

Write each decimal as a fraction.
1. 0.67  
2. 0.9  
3. 0.43

Write each fraction as a decimal.
4. \(\frac{2}{5}\)  
5. \(\frac{1}{6}\)  
6. \(\frac{3}{4}\)

Compare. Write <, >, or =.
7. \(\frac{7}{10}\) 0.9  
8. 0.4 \(\frac{2}{5}\)  
9. \(\frac{3}{5}\) 0.5

10. Jamal got \(\frac{4}{5}\) of the questions correct on his quiz. Dominic got 0.75 of the questions correct. Who got more questions correct?

4-5 Equivalent Fractions

Write two equivalent fractions for each fraction.
11. \(\frac{9}{12}\)  
12. \(\frac{18}{42}\)  
13. \(\frac{25}{30}\)

Write each fraction in simplest form.
14. \(\frac{20}{24}\)  
15. \(\frac{14}{49}\)  
16. \(\frac{12}{28}\)

17. Mandy ate \(\frac{1}{6}\) of a pizza. Write two equivalent fractions for \(\frac{1}{6}\).

18. Liane is making fruit salad. The recipe calls for \(\frac{1}{2}\) cup shredded coconut. Liane has only a \(\frac{1}{4}\)-cup measure. How can she measure the correct \(\frac{1}{2}\)-cup amount?

4-6 Mixed Numbers and Improper Fractions

Replace each shape with a number that will make the equation correct.
19. \(\frac{2}{7} = \frac{9}{?}\)  
20. \(6\frac{1}{8} = \frac{49}{?}\)  
21. \(\frac{4}{9} = \frac{157}{?}\)

Use the table for Exercises 22–24.

<table>
<thead>
<tr>
<th>World’s Longest Movies</th>
<th>Length (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Title</strong></td>
<td><strong>Length</strong></td>
</tr>
<tr>
<td>1900</td>
<td>318 (\frac{5}{60})</td>
</tr>
<tr>
<td>Empire</td>
<td>480 (\frac{5}{60})</td>
</tr>
<tr>
<td>Fanny and Alexander</td>
<td>51 (\frac{5}{5})</td>
</tr>
<tr>
<td>War and Peace</td>
<td>831 (\frac{5}{60})</td>
</tr>
</tbody>
</table>

22. Write the lengths of 1900 and Empire as mixed numbers in simplest form.

23. Write the lengths of Fanny and Alexander and War and Peace as improper fractions.

24. Write the movies in order from longest to shortest.

25. The proboscis bat, with a length of \(\frac{19}{5}\) cm, is one of the smallest bats. Write \(\frac{19}{5}\) as a mixed number.
Write each problem in your own words. Check that you have included all the information you need to answer the question.

1. Martin is making muffins for his class bake sale. The recipe calls for \(2\frac{1}{3}\) cups of flour, but Martin’s only measuring cup holds \(\frac{1}{3}\) cup. How many of his measuring cups should he use?

2. Mariko sold an old book to a used bookstore. She had hoped to sell it for $0.80, but the store gave her \(\frac{3}{4}\) of a dollar. What is the difference between the two amounts?

3. Koalas of eastern Australia feed mostly on eucalyptus leaves. They select certain trees over others to find the \(1\frac{1}{4}\) pounds of food they need each day. Suppose a koala has eaten \(1\frac{1}{8}\) pounds of food. Has the koala eaten enough food for the day?

4. The first day of the Tour de France is called the prologue. Each of the days after that is called a stage, and each stage covers a different distance. The total distance covered in the race is about 3,600 km. If a cyclist has completed \(\frac{1}{3}\) of the race, how many kilometers has he ridden?
Learn to use pictures and number lines to compare and order fractions.

**Vocabulary**
- like fractions
- unlike fractions
- common denominator

Rachel and Hannah are making a kind of cookie called *hamantaschen*. They have \( \frac{1}{2} \) cup of strawberry jam, but the recipe requires \( \frac{1}{3} \) cup.

To determine if they have enough for the recipe, they need to compare the fractions \( \frac{1}{2} \) and \( \frac{1}{3} \).

When you are comparing fractions, first check their denominators. When fractions have the same denominator, they are called **like fractions**. For example, \( \frac{1}{8} \) and \( \frac{5}{8} \) are like fractions. When two fractions have different denominators, they are called **unlike fractions**. For example, \( \frac{7}{10} \) and \( \frac{1}{2} \) are unlike fractions.

**EXAMPLE 1**

**Comparing Fractions**

Compare. Write <, >, or =.

**A**

\[
\frac{1}{8} \quad \square \quad \frac{5}{8}
\]

Model \( \frac{1}{8} \) and \( \frac{5}{8} \).

From the model, \( \frac{1}{8} < \frac{5}{8} \).

**B**

\[
\frac{7}{10} \quad \square \quad \frac{1}{2}
\]

Model \( \frac{7}{10} \) and \( \frac{1}{2} \).

From the model, \( \frac{7}{10} > \frac{1}{2} \).
To compare unlike fractions without models, first rename the fractions so they have the same denominator. This is called finding a common denominator. This method can be used to compare mixed numbers as well.

**Cooking Application**

Rachel and Hannah have 1 \( \frac{2}{3} \) cups of flour. They need 1 \( \frac{1}{2} \) cups to make hamantaschen. Do they have enough flour for the recipe?

Compare 1 \( \frac{2}{3} \) and 1 \( \frac{1}{2} \).

Compare the whole-number parts of the numbers.

1 = 1  The whole-number parts are equal.

Compare the fractional parts. Find a common denominator by multiplying the denominators. 2 \( \cdot \) 3 = 6

Find equivalent fractions with 6 as the denominator.

\[
\begin{align*}
\frac{2}{3} &= \frac{2 \cdot 2}{3 \cdot 2} = \frac{4}{6} \\
\frac{1}{2} &= \frac{1 \cdot 3}{2 \cdot 3} = \frac{3}{6}
\end{align*}
\]

Compare the like fractions. \( \frac{4}{6} > \frac{3}{6} \), so \( \frac{2}{3} > \frac{1}{2} \).

Therefore, 1 \( \frac{2}{3} \) is greater than 1 \( \frac{1}{2} \).

Since 1 \( \frac{2}{3} \) cups is more than 1 \( \frac{1}{2} \) cups, they have enough flour.

**Ordering Fractions**

Order \( \frac{3}{7}, \frac{3}{4}, \) and \( \frac{1}{4} \) from least to greatest.

\[
\begin{align*}
\frac{3}{7} \cdot \frac{4}{4} &= \frac{12}{28} \\
\frac{3}{4} \cdot \frac{7}{7} &= \frac{21}{28} \\
\frac{1}{4} \cdot \frac{7}{7} &= \frac{7}{28}
\end{align*}
\]

Rename with like denominators.

The fractions in order from least to greatest are \( \frac{1}{4}, \frac{3}{7}, \frac{3}{4} \).

**Think and Discuss**

1. Tell whether the values of the fractions change when you rename two fractions so that they have common denominators.

2. Explain how to compare \( \frac{2}{5} \) and \( \frac{4}{5} \).
See Example 1

Compare. Write <, >, or =.

1. \( \frac{3}{5} \quad \frac{2}{5} \)
2. \( \frac{1}{9} \quad \frac{2}{9} \)
3. \( \frac{6}{8} \quad \frac{3}{4} \)
4. \( \frac{3}{7} \quad \frac{6}{7} \)

5. Arsenio has \( \frac{2}{3} \) cup of brown sugar. The recipe he is using requires \( \frac{1}{4} \) cup of brown sugar. Does he have enough brown sugar for the recipe? Explain.

See Example 2

Order the fractions from least to greatest.

6. \( \frac{3}{8} \quad \frac{1}{3} \quad \frac{2}{3} \)
7. \( \frac{1}{4} \quad \frac{2}{5} \quad \frac{3}{7} \)
8. \( \frac{5}{9} \quad \frac{1}{7} \quad \frac{2}{3} \)
9. \( \frac{1}{2} \quad \frac{1}{6} \quad \frac{2}{3} \)

See Example 3

Kelly needs \( \frac{2}{5} \) gallon of paint to finish painting her deck. She has \( \frac{3}{8} \) gallon of paint. Does she have enough paint to finish her deck? Explain.

See Example 3

Order the fractions from least to greatest.

10. \( \frac{2}{5} \quad \frac{4}{5} \)
11. \( \frac{3}{5} \quad \frac{4}{10} \)
12. \( \frac{6}{10} \quad \frac{3}{8} \)
13. \( \frac{5}{6} \quad \frac{4}{6} \)
14. \( \frac{4}{5} \quad \frac{5}{5} \)
15. \( \frac{2}{4} \quad \frac{1}{2} \)
16. \( \frac{4}{8} \quad \frac{16}{24} \)
17. \( \frac{11}{16} \quad \frac{9}{16} \)

See Example 3

Order the fractions from least to greatest.

18. \( \frac{1}{2} \quad \frac{3}{5} \quad \frac{3}{7} \)
19. \( \frac{3}{5} \quad \frac{9}{10} \quad \frac{3}{7} \)
20. \( \frac{1}{2} \quad \frac{2}{5} \quad \frac{1}{4} \)
21. \( \frac{4}{9} \quad \frac{3}{8} \quad \frac{1}{3} \)
22. \( \frac{1}{4} \quad \frac{5}{6} \quad \frac{5}{9} \)
23. \( \frac{3}{10} \quad \frac{2}{5} \quad \frac{4}{3} \)
24. \( \frac{13}{5} \quad \frac{5}{18} \quad \frac{9}{6} \)
25. \( \frac{3}{1} \quad \frac{2}{8} \quad \frac{3}{4} \)
26. \( \frac{1}{5} \quad \frac{5}{6} \quad \frac{5}{9} \)

Extra Practice

See page 721.

Compare. Write <, >, or =.

27. \( \frac{4}{15} \quad \frac{3}{10} \)
28. \( \frac{7}{12} \quad \frac{13}{30} \)
29. \( \frac{5}{9} \quad \frac{4}{11} \)
30. \( \frac{8}{14} \quad \frac{8}{9} \)
31. \( \frac{3}{5} \quad \frac{26}{65} \)
32. \( \frac{3}{5} \quad \frac{2}{21} \)
33. \( \frac{24}{41} \quad \frac{2}{7} \)
34. \( \frac{10}{38} \quad \frac{1}{4} \)

Order the fractions from least to greatest.

35. \( \frac{2}{5} \quad \frac{3}{10} \quad \frac{2}{2} \)
36. \( \frac{3}{7} \quad \frac{3}{4} \quad \frac{5}{10} \)
37. \( \frac{7}{15} \quad \frac{2}{3} \quad \frac{1}{5} \)
38. \( \frac{3}{4} \quad \frac{1}{3} \quad \frac{8}{15} \)
39. \( \frac{2}{4} \quad \frac{11}{9} \quad \frac{15}{12} \)
40. \( \frac{5}{12} \quad \frac{1}{8} \quad \frac{1}{2} \)
41. \( \frac{5}{3} \quad \frac{5}{8} \quad \frac{12}{4} \)
42. \( \frac{2}{3} \quad \frac{7}{8} \quad \frac{15}{15} \)

43. Laura and Kim receive the same amount of allowance each week. Laura spends \( \frac{2}{3} \) of it on going to the movies. Kim spends \( \frac{1}{2} \) of it on a CD. Which girl spent more of her allowance? Explain.

44. Kyle operates a hot dog cart in a large city. He spends \( \frac{2}{5} \) of his budget on supplies, \( \frac{1}{12} \) on advertising, and \( \frac{2}{25} \) on taxes and fees. Does Kyle spend more on advertising or more on taxes and fees?
Order the numbers from least to greatest.

45. \(1 \frac{2}{5}, 1 \frac{1}{8}, 3 \frac{4}{5}, 3, 3 \frac{2}{5}\)  

46. \(7 \frac{1}{2}, 9 \frac{4}{7}, 9 \frac{1}{2}, 8, 8 \frac{3}{4}\)  

47. \(\frac{1}{2}, \frac{3}{5}, 3 \frac{1}{10}, \frac{3}{4}, \frac{3}{15}\)

48. **Agriculture** The table shows the fraction of the world's total corn each country produces. List the countries in order from the country that produces the most corn to the country that produces the least corn.

<table>
<thead>
<tr>
<th>World's Corn Production</th>
</tr>
</thead>
<tbody>
<tr>
<td>United States</td>
</tr>
<tr>
<td>China</td>
</tr>
<tr>
<td>Brazil</td>
</tr>
</tbody>
</table>

49. **Multi-Step** The Dixon Dragons must win at least \(\frac{3}{7}\) of their remaining games to qualify for their district playoffs. If they have 15 games left and they win 7 of them, will the Dragons compete in the playoffs? Explain.

50. **Write a Problem** Write a problem that involves comparing two fractions with different denominators.

51. **Write About It** Compare the following fractions.

\[\begin{array}{cccc}
\frac{1}{2} & \frac{1}{4} & \frac{2}{3} & \frac{2}{5} \\
\frac{3}{4} & \frac{3}{7} & \frac{4}{5} & \frac{4}{9}
\end{array}\]

What do you notice about two fractions that have the same numerator but different denominators? Which one is greater?

52. **Challenge** Name a fraction that would make the inequality true.

\[\frac{1}{4} > \_ > \frac{1}{5}\]

---

53. **Multiple Choice** Which fraction has the least value?

A. \(\frac{1}{5}\)  
B. \(\frac{3}{11}\)  
C. \(\frac{2}{15}\)  
D. \(\frac{4}{18}\)

54. **Extended Response** Kevin is making potato soup. The recipe shows that he needs \(\frac{1}{2}\) gallon of milk and 3.5 pounds of potatoes. He has \(\frac{5}{2}\) gallon of milk and \(\frac{21}{5}\) pounds of potatoes. Does Kevin have enough milk and potatoes to make the soup? Show your work and explain your answer.

Write each number in scientific notation. (Lesson 3-4)

55. 45  
56. 405,000  
57. 1,600,000  
58. 23,000,000

Write each fraction in simplest form. (Lesson 4-5)

59. \(\frac{3}{36}\)  
60. \(\frac{4}{42}\)  
61. \(\frac{6}{20}\)  
62. \(\frac{12}{30}\)  
63. \(\frac{5}{55}\)
Learn to add and subtract fractions with like denominators.

You can estimate the age of an oak tree by measuring around the trunk at four feet above the ground.

The distance around a young oak tree’s trunk increases at a rate of approximately \( \frac{1}{8} \) inch per month.

**Example 1**

**Life Science Application**

Sophie plants a young oak tree in her backyard. The distance around the trunk grows at a rate of \( \frac{1}{8} \) inch per month. Use pictures to model how much this distance will increase in two months, then write your answer in simplest form.

\[
\begin{align*}
\frac{1}{8} + \frac{1}{8} &= \frac{2}{8} \\
&= \frac{1}{4}
\end{align*}
\]

Add the numerators. Keep the same denominator.

Write your answer in simplest form.

The distance around the trunk will increase by \( \frac{1}{4} \) inch.

**Example 2**

**Subtracting Like Fractions and Mixed Numbers**

Subtract. Write each answer in simplest form.

**A** \( 1 - \frac{2}{3} \)

To get a common denominator, rewrite 1 as a fraction with a denominator of 3.

\[
\frac{3}{3} - \frac{2}{3} = \frac{1}{3}
\]

Subtract the numerators. Keep the same denominator.

Check

\[
\begin{align*}
\text{---} & - \text{---} = \text{---}
\end{align*}
\]
Think and Discuss

1. Explain how to add or subtract like fractions.

2. Tell why the sum of \(\frac{1}{3}\) and \(\frac{3}{5}\) is not \(\frac{4}{10}\). Give the correct sum.

3. Describe how you would add \(\frac{2}{3}\) and \(\frac{1}{8}\). How would you subtract \(1\frac{1}{8}\) from \(2\frac{3}{8}\)?
**Guided Practice**

1. Marta is filling a bucket with water. The height of the water is increasing \( \frac{1}{6} \) foot each minute. Use pictures to model how much the height of the water will change in three minutes, and then write your answer in simplest form.

2. Subtract. Write each answer in simplest form.
   - \( 2 - \frac{3}{5} \)
   - \( 3 - \frac{6}{7} \)
   - \( 4 \frac{2}{3} - 1 \frac{1}{3} \)
   - \( 8 \frac{7}{12} - 3 \frac{5}{12} \)

3. Evaluate each expression for \( x = \frac{3}{10} \). Write each answer in simplest form.
   - \( \frac{9}{10} - x \)
   - \( x + \frac{1}{10} \)
   - \( x + \frac{9}{10} \)
   - \( x - \frac{1}{10} \)

**Independent Practice**

4. Wesley drinks \( \frac{2}{13} \) gallon of juice each day. Use pictures to model the number of gallons of juice Wesley drinks in 5 days, and then write your answer in simplest form.

5. Subtract. Write each answer in simplest form.
   - \( 1 - \frac{5}{7} \)
   - \( 1 - \frac{3}{8} \)
   - \( 2 \frac{4}{5} - 1 \frac{1}{5} \)
   - \( 9 \frac{9}{14} - 5 \frac{3}{14} \)

6. Evaluate each expression for \( x = \frac{11}{20} \). Write each answer in simplest form.
   - \( x + \frac{13}{20} \)
   - \( x - \frac{3}{20} \)
   - \( x - \frac{9}{20} \)
   - \( x + \frac{17}{20} \)

**Practice and Problem Solving**

Write each sum or difference in simplest form.

- \( \frac{1}{16} + \frac{9}{16} \)
- \( \frac{15}{26} - \frac{11}{26} \)
- \( \frac{10}{33} + \frac{4}{33} \)
- \( 1 - \frac{9}{10} \)
- \( \frac{26}{75} + \frac{24}{75} \)
- \( 100 \frac{999}{999} + 899 \frac{999}{999} \)
- \( 37 \frac{13}{18} - 24 \frac{7}{18} \)
- \( \frac{1}{20} + \frac{7}{20} + \frac{3}{20} \)
- \( \frac{11}{24} + \frac{1}{24} + \frac{5}{24} \)

28. Lily took \( \frac{5}{8} \) lb of peanuts to a baseball game. She ate \( \frac{2}{5} \) lb. How many pounds of peanuts does she have left? Write the answer in simplest form.

Evaluate. Write each answer in simplest form.

- \( a + \frac{7}{18} \) for \( a = \frac{1}{18} \)
- \( \frac{6}{13} - j \) for \( j = \frac{4}{13} \)
- \( c + c \) for \( c = \frac{5}{14} \)
- \( m - \frac{6}{17} \) for \( m = 1 \)
- \( 8 \frac{14}{15} - z \) for \( z = \frac{4}{15} \)
- \( 13 \frac{1}{24} + y \) for \( y = 2 \frac{5}{24} \)

35. Sheila spent \( x \) hour studying on Tuesday and \( \frac{1}{4} \) hour studying on Thursday. What was the total amount of time in hours Sheila spent studying if \( x = \frac{27}{4} \)?
36. Carlos had 7 cups of chocolate chips. He used \(1 \frac{2}{3}\) cups to make a chocolate sauce and \(3 \frac{3}{5}\) cups to make cookies. How many cups of chocolate chips does Carlos have now?

37. A concert was \(2 \frac{3}{4}\) hr long. The first musical piece lasted \(1 \frac{1}{4}\) hr. The intermission also lasted \(1 \frac{1}{4}\) hr. How long was the rest of the concert?

38. A flight from Washington, D.C., stops in San Francisco and then continues to Seattle. The trip to San Francisco takes \(4 \frac{5}{8}\) hr. The trip to Seattle takes \(1 \frac{1}{8}\) hr. What is the total flight time?

**Life Science**  Use the graph for Exercises 39–41. Sheila performed an experiment to find the most effective plant fertilizer. She used a different fertilizer on each of 5 different plants. The heights of the plants at the end of her experiment are shown in the graph.

39. What is the combined height of plants C and E?

40. What is the difference in height between the tallest plant and the shortest plant?

41. **What’s the Error?** Sheila found the combined heights of plants B and E to be \(1 \frac{5}{24}\) feet. Explain the error and give the correct answer in simplest form.

42. **Write About It** When writing 1 as a fraction in a subtraction problem, how do you know what the numerator and denominator should be? Give an example.

43. **Challenge** Explain how you might estimate the difference between \(\frac{3}{4}\) and \(\frac{6}{23}\).

---

**Test Prep and Spiral Review**

44. **Multiple Choice** Solve. \(x - \frac{6}{11} = \frac{5}{11}\)
   
   A \(\frac{1}{22}\)  
   B \(\frac{1}{11}\)  
   C 1  
   D 11

45. **Short Response** Your friend was absent from school and asked you for help with the math assignment. Give your friend detailed instructions on how to subtract \(4 \frac{7}{12}\) from \(13 \frac{11}{12}\).

Find two equivalent fractions for each fraction. (Lesson 4-5)

46. \(\frac{4}{7}\)  
47. \(\frac{3}{4}\)  
48. \(\frac{2}{9}\)  
49. \(\frac{3}{5}\)  
50. \(\frac{1}{10}\)

Order the fractions from least to greatest. (Lesson 4-7)

51. \(\frac{3}{7}, \frac{5}{4}, \frac{2}{6}\)  
52. \(\frac{2}{3}, \frac{4}{11}, \frac{5}{8}\)  
53. \(\frac{3}{10}, \frac{3}{8}, \frac{1}{3}\)
Learn to estimate sums and differences of fractions and mixed numbers.

Members of the Nature Club went mountain biking in Canyonlands National Park, Utah. They biked $10\frac{3}{10}$ miles on Monday.

You can estimate fractions by rounding to 0, $\frac{1}{2}$, or 1.

The fraction $\frac{3}{4}$ is halfway between $\frac{1}{2}$ and 1, but we usually round up. So the fraction $\frac{3}{4}$ rounds to 1.

You can round fractions by comparing the numerator and denominator.

**EXAMPLE 1**

Estimating Fractions

Estimate each sum or difference by rounding to 0, $\frac{1}{2}$, or 1.

**A**

$\frac{8}{9} + \frac{2}{11}$

Think: $\frac{8}{9}$ rounds to 1 and $\frac{2}{11}$ rounds to 0.

$1 + 0 = 1$

$\frac{8}{9} + \frac{2}{11}$ is about 1.

**B**

$\frac{7}{12} - \frac{8}{15}$

Think: $\frac{7}{12}$ rounds to $\frac{1}{2}$ and $\frac{8}{15}$ rounds to $\frac{1}{2}$.

$\frac{1}{2} - \frac{1}{2} = 0$

$\frac{7}{12} - \frac{8}{15}$ is about 0.
You can also estimate by rounding mixed numbers. Compare the mixed number to the two nearest whole numbers and the nearest \( \frac{1}{2} \).

Does \( 10\frac{3}{10} \) round to 10, \( 10\frac{1}{2} \), or 11?

The mixed number \( 10\frac{3}{10} \) rounds to \( 10\frac{1}{2} \).

**EXAMPLE 2**

**Sports Application**

**A** About how far did the Nature Club ride on Monday and Tuesday?

\[
10\frac{3}{10} + 9\frac{3}{4}
\]

\[
10\frac{1}{2} + 10 = 20\frac{1}{2}
\]

They rode about 20\( \frac{1}{2} \) miles.

**B** About how much farther did the Nature Club ride on Wednesday than on Thursday?

\[
12\frac{1}{4} - 4\frac{7}{10}
\]

\[
12\frac{1}{2} - 4\frac{1}{2} = 8
\]

They rode about 8 miles farther on Wednesday than on Thursday.

**C** Estimate the total distance that the Nature Club rode on Monday, Tuesday, and Wednesday.

\[
10\frac{3}{10} + 9\frac{3}{4} + 12\frac{1}{4}
\]

\[
10\frac{1}{2} + 10 + 12\frac{1}{2} = 33
\]

They rode about 33 miles.

**Think and Discuss**

1. **Tell** whether each fraction rounds to 0, \( \frac{1}{2} \), or 1: \( \frac{5}{6} \), \( \frac{2}{15} \), \( \frac{7}{13} \).

2. **Explain** how to round mixed numbers to the nearest whole number.

3. **Determine** whether the Nature Club met their goal to ride at least 35 total miles.
Estimate each sum or difference by rounding to 0, ½, or 1.

1. \( \frac{8}{9} + \frac{1}{6} \)
2. \( \frac{11}{12} - \frac{4}{9} \)
3. \( \frac{3}{7} + \frac{1}{12} \)
4. \( \frac{6}{13} - \frac{2}{5} \)

Use the table for Exercises 5 and 6.

5. About how far did Mark run during week 1 and week 2?
6. About how much farther did Mark run during week 2 than during week 3?

Mark’s Running Distances

<table>
<thead>
<tr>
<th>Week</th>
<th>Distance (mi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>8(\frac{3}{4})</td>
</tr>
<tr>
<td>2</td>
<td>7(\frac{1}{5})</td>
</tr>
<tr>
<td>3</td>
<td>5(\frac{5}{6})</td>
</tr>
</tbody>
</table>

Use the table for Exercises 15–17.

15. About how much do the meteorites in Brenham and Goose Lake weigh together?
16. About how much more does the meteorite in Willamette weigh than the meteorite in Norton County?
17. About how much do the two meteorites in Kansas weigh together?

Meteorites in the United States

<table>
<thead>
<tr>
<th>Location</th>
<th>Weight (tons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Willamette, AZ</td>
<td>16(\frac{1}{2})</td>
</tr>
<tr>
<td>Brenham, KS</td>
<td>2(\frac{3}{5})</td>
</tr>
<tr>
<td>Goose Lake, CA</td>
<td>1(\frac{3}{10})</td>
</tr>
<tr>
<td>Norton County, KS</td>
<td>1(\frac{1}{10})</td>
</tr>
</tbody>
</table>

Estimate each sum or difference to compare. Write < or >.

18. \( \frac{5}{6} + \frac{7}{9} \)
19. \( \frac{28}{15} - \frac{11}{11} \)
20. \( \frac{12}{21} + \frac{3}{7} \)
21. \( \frac{7}{13} - \frac{8}{9} \)
22. \( \frac{32}{10} + \frac{22}{5} \)
23. \( \frac{46}{9} - \frac{23}{19} \)
24. Critical Thinking Describe a situation in which it is better to round a mixed number up to the next whole number even if the fraction in the mixed number is closer to \( \frac{1}{2} \) than 1.

Estimate.

25. \( \frac{7}{8} + \frac{4}{7} + \frac{7}{13} \)
26. \( \frac{6}{11} + \frac{9}{17} + \frac{3}{5} \)
27. \( \frac{8}{9} + \frac{3}{4} + \frac{9}{10} \)
28. \( \frac{1\frac{5}{8}}{12} + \frac{2\frac{1}{15}}{12} + \frac{2\frac{12}{13}}{12} \)
29. \( \frac{4\frac{11}{12}}{19} + \frac{3\frac{1}{19}}{19} + \frac{5\frac{4}{7}}{19} \)
30. \( \frac{10\frac{1}{9}}{14} + \frac{8\frac{5}{9}}{14} + \frac{11\frac{13}{14}}{14} \)
31. About how much longer is the harlequin beetle than the cetonid beetle?

32. About how much longer is the harlequin beetle than the chrysomeliad beetle?

33. Use the table to estimate the total weekly snowfall.

<table>
<thead>
<tr>
<th>Day</th>
<th>Mon</th>
<th>Tue</th>
<th>Wed</th>
<th>Thu</th>
<th>Fri</th>
<th>Sat</th>
<th>Sun</th>
</tr>
</thead>
<tbody>
<tr>
<td>Snowfall (in.)</td>
<td>$\frac{34}{7}$</td>
<td>$\frac{7}{8}$</td>
<td>0</td>
<td>$\frac{21}{6}$</td>
<td>$\frac{2}{11}$</td>
<td>$\frac{9}{20}$</td>
<td>$\frac{14}{7}$</td>
</tr>
</tbody>
</table>

34. Write a Problem Write a problem about a trip that can be solved by estimating fractions. Exchange with a classmate and solve.

35. Write About It Explain how to estimate the sum of two mixed numbers. Give an example to explain your answer.

36. Challenge Estimate. $\left[\frac{5}{8} - \frac{3}{20}\right] + \frac{14}{7}$

---

**Test Prep and Spiral Review**

37. Multiple Choice Larry ran $3\frac{1}{2}$ miles on Monday and $5\frac{3}{4}$ miles on Tuesday. About how many miles did Larry run on Monday and Tuesday?

- A 8
- B 9
- C 10
- D 11

38. Multiple Choice Marie used $2\frac{2}{3}$ cups of flour for a recipe. Linda used $1\frac{1}{4}$ cups of flour for a recipe. About how many more cups of flour did Marie use than Linda?

- F 1
- G 2
- H 3
- I 4

Evaluate each expression. (Lesson 1-4)

39. $6 \times (21 - 15) \div 12$

40. $72 \div 8 + 2^3 \times 5 - 19$

41. $5 + (6 - 1) \times 2 \div 2$

Write an expression for the missing value in each table. (Lesson 2-3)

42.

<table>
<thead>
<tr>
<th>Games Played</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Points Scored</td>
<td>14</td>
<td>28</td>
<td>42</td>
<td>56</td>
<td></td>
</tr>
</tbody>
</table>

43.

<table>
<thead>
<tr>
<th>Month</th>
<th>1</th>
<th>3</th>
<th>5</th>
<th>7</th>
<th>n</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hours Worked</td>
<td>6</td>
<td>8</td>
<td>10</td>
<td>12</td>
<td></td>
</tr>
</tbody>
</table>
Quiz for Lessons 4-7 Through 4-9

4-7 Comparing and Ordering Fractions

Compare. Write <, >, or =.

1. \(\frac{3}{4} \quad \frac{2}{3}\)  
2. \(\frac{7}{9} \quad \frac{5}{6}\)  
3. \(\frac{4}{9} \quad \frac{4}{7}\)  
4. \(\frac{5}{11} \quad \frac{3}{5}\)

Order the fractions from least to greatest.

5. \(\frac{5}{8} \quad \frac{1}{2} \quad \frac{3}{4}\)  
6. \(\frac{3}{4} \quad \frac{3}{5} \quad \frac{7}{10}\)  
7. \(\frac{1}{3} \quad \frac{3}{8} \quad \frac{1}{4}\)  
8. \(\frac{2}{5} \quad \frac{9}{11}\)

9. Mrs. Wilson split a bag of marbles between her three sons. Ralph got \(\frac{1}{10}\), Pete got \(\frac{1}{2}\), and Jon got \(\frac{8}{20}\). Who got the most marbles?

4-8 Adding and Subtracting with Like Denominators

10. The average growth rate for human hair is \(\frac{1}{2}\) inch per month. On average, how much hair will a person grow in 3 months? Write your answer in simplest form.

11. A recipe for fruit salad calls for \(\frac{1}{5}\) cup coconut. Ryan wants to double the recipe. How much coconut should he use? Write your answer in simplest form.

Subtract. Write each answer in simplest form.

12. \(1 - \frac{3}{4}\)  
13. \(6\frac{5}{9} - \frac{5}{1}{9}\)  
14. \(10\frac{7}{16} - 4\frac{3}{16}\)

15. \(\frac{8}{9} - \frac{7}{9}\)  
16. \(8\frac{4}{17} - 6\frac{2}{17}\)  
17. \(1 - \frac{7}{8}\)

Evaluate each expression for \(x = \frac{5}{7}\). Write your answer in simplest form.

18. \(x + 2\frac{1}{7}\)  
19. \(x - \frac{3}{7}\)  
20. \(10 + x\)

21. \(3\frac{2}{7} + x\)  
22. \(6\frac{6}{7} - x\)  
23. \(x + \frac{2}{7}\)

4-9 Estimating Fraction Sums and Differences

Estimate each sum or difference.

24. \(\frac{3}{4} - \frac{1}{10}\)  
25. \(\frac{7}{9} + \frac{7}{9}\)  
26. \(\frac{15}{16} - \frac{4}{5}\)  
27. \(\frac{3}{7} \frac{1}{10}\)

Use the table for problems 28–30.

<table>
<thead>
<tr>
<th>Mrs. Ping’s Walking Distances</th>
</tr>
</thead>
<tbody>
<tr>
<td>Week</td>
</tr>
<tr>
<td>------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
</tbody>
</table>

28. About how far did Mrs. Ping walk during week 1 and week 2?

29. About how much farther did Mrs. Ping walk in week 2 than in week 3?

30. About how far did Mrs. Ping walk during the three weeks?
A Party with Palm Trees
Jamal and Sarah are planning an end-of-year party for the Spanish Club. They want it to have a tropical theme.

1. There will be 16 girls and 12 boys at the party. Jamal wants to set up the tables so that every table has the same number of girls and the same number of boys. How many tables will there be? How many girls and boys will be at each table?

2. Sarah finds three recipes for fruit punch. She wants to choose the recipe that calls for the greatest amount of pineapple juice per serving. Which recipe should she choose? Explain.

3. Jamal thinks they should choose the recipe that makes the largest serving of punch. Which recipe should they choose in this case? Explain.

4. Each punch glass holds 2 cups of liquid. If they use the recipe that makes the largest serving, will there be room in each glass for ice? Explain.
Sets of Numbers

A group of items is called a **set**. The items in a set are called **elements**. In this chapter, you saw several sets of numbers, such as prime numbers, composite numbers, and factors.

In a **Venn diagram**, circles are used to show relationships between sets. The overlapped region represents elements that are in both set $A$ and set $B$. This set is called the **intersection** of $A$ and $B$. Elements that are in set $A$ or set $B$ make up the **union** of $A$ and $B$.

**Example 1**

Identify the elements in each set. Then draw a Venn diagram. What is the intersection? What is the union?

<table>
<thead>
<tr>
<th>Set A: prime numbers</th>
<th>Set B: composite numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elements of A: 2, 3, 5, 7, …</td>
<td>Elements of B: 4, 6, 8, 9, …</td>
</tr>
</tbody>
</table>

**A**

Intersection: none. When a set has no elements, it is called an **empty set**. The intersection of $A$ and $B$ is empty.

Union: all numbers that are prime or composite—all whole numbers except 0 and 1.

<table>
<thead>
<tr>
<th>Set A: factors of 36</th>
<th>Set B: factors of 24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elements of A: 1, 2, 3, 4, 6, 9, 12, 18, 36</td>
<td>Elements of B: 1, 2, 3, 4, 6, 8, 12, 24</td>
</tr>
</tbody>
</table>

**B**

The circles overlap because some factors of 36 are also factors of 24.

Intersection: 1, 2, 3, 4, 6, 12

Union: 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36

**Vocabulary**
- set
- empty set
- element
- subset
- intersection
- union
- Venn diagram
Identify the elements in each set. Then draw a Venn diagram. What is the intersection? What is the union?

C Set A: factors of 36  Set B: factors of 12
Elements of A: 1, 2, 3, 4, 6, 9, 12, 18, 36
Elements of B: 1, 2, 3, 4, 6, 12

Intersection: 1, 2, 3, 4, 6, 12
Union: 1, 2, 3, 4, 6, 9, 12, 18, 36

Look at Example 1C. When one set is entirely contained in another set, we say the first set is a subset of the second set.

Exercise 1: Set A: even numbers  Set B: odd numbers
Exercise 2: Set A: factors of 18  Set B: factors of 40
Exercise 3: Set A: factors of 72  Set B: factors of 36
Exercise 4: Set A: even numbers  Set B: composite numbers

Tell whether set A is a subset of set B.

Exercise 5: Set A: whole numbers less than 10  Set B: whole numbers less than 12
Exercise 6: Set A: whole numbers less than 8  Set B: whole numbers greater than 9
Exercise 7: Set A: prime numbers  Set B: odd numbers
Exercise 8: Set A: numbers divisible by 6  Set B: numbers divisible by 3

Write About It How could you use a Venn diagram to help find the greatest common factor of two numbers? Give an example.

Challenge How could you use a Venn diagram to help find the greatest common factor of three numbers? Give an example.
Riddle Me This

“When you go from there to here, you’ll find I disappear. Go from here to there, and then you’ll see me again. What am I?”

To solve this riddle, copy the square below. If a number is divisible by 3, color that box red. Remember the divisibility rule for 3. If a number is not divisible by 3, color that box blue.

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>102</td>
<td>981</td>
<td>210</td>
<td>6,015</td>
<td>72</td>
</tr>
<tr>
<td>79</td>
<td>1,204</td>
<td>576</td>
<td>10,019</td>
<td>1,771</td>
</tr>
<tr>
<td>548</td>
<td>3,416</td>
<td>12,300</td>
<td>904</td>
<td>1,330</td>
</tr>
<tr>
<td>217</td>
<td>2,662</td>
<td>1,746</td>
<td>3,506</td>
<td>15,025</td>
</tr>
<tr>
<td>34,351</td>
<td>725</td>
<td>2,352</td>
<td>5,675</td>
<td>6,001</td>
</tr>
</tbody>
</table>

On a Roll

The object is to be the first person to fill in all the squares on your game board.

On your turn, roll a number cube and record the number rolled in any blank square on your game board. Once you have placed a number in a square, you cannot move that number. If you cannot place the number in a square, then your turn is over. The winner is the first player to complete their game board correctly.

A complete copy of the rules and game pieces are available online.
**PROJECT** Spec-Tag-Ular Number Theory

Tags will help you keep notes about number theory and fractions on an easy-to-use reference ring.

**Directions**

1. Make tags by cutting ten rectangles from card stock, each approximately $2\frac{3}{4}$ inches by $1\frac{1}{2}$ inches.

2. Use scissors to clip off two corners at the end of each tag. Figure A

3. Punch a hole between the clipped corners of each tag. Put a reinforcement around the hole on both sides of the tag. Figure B

4. Hook all of the tags together on a wire ring. On one of the tags, write the number and name of the chapter. Figure C

**Taking Note of the Math**

On each tag, write a divisibility rule from the chapter. You can also use the tags to record important facts about fractions.
Vocabulary

common denominator .................................. 199
composite number ...................................... 165
divisible .................................................. 164
equivalent fractions .................................... 186
factor ....................................................... 169
greatest common factor (GCF) ................. 173
improper fraction ..................................... 192
like fractions ............................................. 198
mixed number .......................................... 181
prime factorization ..................................... 169
prime number .......................................... 165
proper fraction .......................................... 192
repeating decimal ..................................... 192
simplest form ............................................ 182
terminating decimal ................................... 182
unlike fractions ......................................... 198

Complete the sentences below with vocabulary words from the list above.

1. The number \(\frac{11}{5}\) is an example of a(n) \(\underline{\text{?}}\), and \(3\frac{1}{6}\) is an example of a(n) \(\underline{\text{?}}\).

2. A(n) \(\underline{\text{?}}\), such as 0.3333..., has a block of one or more digits that repeat without end. A(n) \(\underline{\text{?}}\), such as 0.25, has a finite number of decimal places.

3. A(n) \(\underline{\text{?}}\) is divisible by only two numbers, 1 and itself. A(n) \(\underline{\text{?}}\) is divisible by more than two numbers.

### 4-1 Divisibility (pp. 164–167)

**Example**

- Tell whether 210 is divisible by 2, 3, 4, and 6.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>The last digit, 0, is even.</td>
<td>Divisible</td>
</tr>
<tr>
<td>3</td>
<td>The sum of the digits is divisible by 3.</td>
<td>Divisible</td>
</tr>
<tr>
<td>4</td>
<td>The number formed by the last two digits is not divisible by 4.</td>
<td>Not divisible</td>
</tr>
<tr>
<td>6</td>
<td>210 is divisible by 2 and 3.</td>
<td>Divisible</td>
</tr>
</tbody>
</table>

- Tell whether each number is prime or composite.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>17</td>
<td>only divisible by 1 and 17 (\underline{\text{prime}})</td>
</tr>
<tr>
<td>25</td>
<td>divisible by 1, 5, and 25 (\underline{\text{composite}})</td>
</tr>
</tbody>
</table>

**Exercises**

Tell whether each number is divisible by 2, 3, 4, 5, 6, 9, and 10.

4. 118
6. 342
8. 170
9. 393

Tell whether each number is prime or composite.

10. 121
12. 13
14. 67
16. 39
18. 85
11. 77
13. 118
15. 93
17. 97
19. 61
4-2 Factors and Prime Factorization (pp. 169–172)

**Example**

- List all the factors of 10.
  
  $10 = 1 \cdot 10$  
  $10 = 2 \cdot 5$

  The factors of 10 are 1, 2, 5, and 10.

- Write the prime factorization of 30.
  
  $30 = 2 \cdot 3 \cdot 5$

**Exercises**

List all the factors of each number.

20. 60  
21. 72  
22. 29  
23. 56  
24. 85  
25. 71

Write the prime factorization of each number.

26. 65  
27. 94  
28. 110  
29. 81  
30. 99  
31. 76

4-3 Greatest Common Factor (pp. 173–176)

**Example**

- Find the GCF of 35 and 50.
  
  factors of 35: 1, 5, 7, 35  
  factors of 50: 1, 2, 5, 10, 25, 50  

  The GCF of 35 and 50 is 5.

**Exercises**

Find the GCF of each set of numbers.

35. 36 and 60  
36. 50, 75, and 125  
37. 45, 81, and 99

4-4 Decimals and Fractions (pp. 181–184)

**Example**

- Write 1.29 as a mixed number.
  
  $1.29 = 1\frac{29}{100}$

- Write $\frac{3}{5}$ as a decimal.
  
  $0.6$  
  $\frac{3}{5} = 0.6$

**Exercises**

Write as a fraction or mixed number.

38. 0.37  
39. 1.8  
40. 0.4

Write as a decimal.

41. $\frac{7}{8}$  
42. $\frac{2}{5}$  
43. $\frac{7}{9}$

4-5 Equivalent Fractions (pp. 186–189)

**Example**

- Find an equivalent fraction for $\frac{4}{5}$.
  
  $\frac{4}{5} = \frac{4 \cdot 3}{5 \cdot 3} = \frac{12}{15}$

- Write $\frac{8}{12}$ in simplest form.
  
  $\frac{8}{12} = \frac{2}{3}$

**Exercises**

Find two equivalent fractions.

44. $\frac{4}{6}$  
45. $\frac{4}{5}$  
46. $\frac{3}{12}$

Write each fraction in simplest form.

47. $\frac{14}{16}$  
48. $\frac{9}{30}$  
49. $\frac{7}{10}$
4-6 Mixed Numbers and Improper Fractions (pp. 192–195)

**Example**
- Write $3\frac{5}{6}$ as an improper fraction.
  $$3\frac{5}{6} = \frac{(3 \cdot 6) + 5}{6} = \frac{18 + 5}{6} = \frac{23}{6}$$

- Write $\frac{19}{4}$ as a mixed number.
  $$4R3 \quad \frac{19}{4} = 4\frac{3}{4}$$

**Exercises**
- Write as an improper fraction.
  50. $\frac{7}{9}$
  51. $\frac{5}{12}$
  52. $\frac{2}{7}$
- Write as a mixed number.
  53. $\frac{23}{6}$
  54. $\frac{17}{5}$
  55. $\frac{41}{8}$

4-7 Comparing and Ordering Fractions (pp. 198–201)

**Example**
- Order from least to greatest.
  Rename with like denominators.
  $$\frac{3}{5}, \frac{2}{3}, \frac{1}{3}, \frac{1}{5} \quad \frac{3 \cdot 15}{5 \cdot 15} = \frac{45}{15}, \frac{2 \cdot 5}{3 \cdot 5} = \frac{10}{15}, \frac{1 \cdot 3}{3 \cdot 3} = \frac{3}{9}$$

**Exercises**
- Compare. Write $<$, $>$, or $=$.
  56. $\frac{6}{8} \underline{\quad} \frac{3}{8}$
  57. $\frac{7}{9} \underline{\quad} \frac{2}{3}$
- Order from least to greatest.
  58. $\frac{3}{8}, \frac{2}{3}, \frac{7}{8}$
  59. $\frac{4}{6}, \frac{3}{12}, \frac{1}{3}$

4-8 Adding and Subtracting with Like Denominators (pp. 202–205)

**Example**
- Subtract $4\frac{5}{6} - 2\frac{1}{6}$. Write your answer in simplest form.
  $$4\frac{5}{6} - 2\frac{1}{6} = 2\frac{2}{6} = 2\frac{1}{3}$$

**Exercises**
- Add or subtract. Write each answer in simplest form.
  60. $\frac{1}{5} + \frac{4}{5}$
  61. $1 - \frac{3}{12}$
  62. $\frac{9}{10} - \frac{3}{10}$
  63. $\frac{2}{7} + 2\frac{3}{7}$

4-9 Estimating Fraction Sums and Differences (pp. 206–209)

**Example**
- Estimate the sum or difference by rounding fractions to 0, $\frac{1}{2}$, or 1.
  $$\frac{7}{8} + \frac{1}{7} \quad \text{Think:} \ 1 + 0.$$  
  $$\frac{7}{8} + \frac{1}{7} \text{ is about 1.}$$

**Exercises**
- Estimate each sum or difference by rounding fractions to 0, $\frac{1}{2}$, or 1.
  64. $\frac{3}{5} + \frac{3}{7}$
  65. $\frac{6}{7} - \frac{5}{9}$
  66. $\frac{49}{10} + 6\frac{1}{5}$
  67. $7\frac{5}{11} - 4\frac{3}{4}$
List all the factors of each number. Then tell whether each number is prime or composite.
1. 98  2. 40  3. 45

Write the prime factorization of each number.
4. 64  5. 130  6. 49

Find the GCF of each set of numbers.
7. 24 and 108  8. 45, 18, and 39  9. 49, 77, and 84

10. Ms. Arrington is making supply boxes for her students. She has 63 pencils, 42 pens, and 21 packs of markers. Each type of supply must be evenly distributed. What is the greatest number of supply boxes she can make if every supply is used?

Write each decimal as a fraction or mixed number.
11. 0.37  12. 1.9  13. 0.92

Write each fraction or mixed number as a decimal.
14. \(\frac{3}{8}\)  15. \(9\frac{3}{5}\)  16. \(\frac{2}{3}\)

Write each fraction in simplest form.
17. \(\frac{4}{12}\)  18. \(\frac{6}{9}\)  19. \(\frac{3}{15}\)

Write each mixed number as an improper fraction.
20. \(4\frac{7}{8}\)  21. \(7\frac{5}{12}\)  22. \(3\frac{5}{7}\)

Compare. Write <, >, or =.
23. \(\frac{5}{6} \square \frac{3}{6}\)  24. \(\frac{3}{4} \square \frac{7}{8}\)  25. \(\frac{4}{5} \square \frac{7}{10}\)

Order the fractions and decimals from least to greatest.
26. 2.17, 2.3, \(\frac{21}{9}\)  27. 0.1, \(\frac{3}{8}\), 0.3  28. 0.9, \(\frac{2}{8}\), 0.35

29. On Monday, it snowed \(2\frac{1}{4}\) inches. On Tuesday, an additional \(3\frac{3}{4}\) inches of snow fell. How much snow fell altogether on Monday and Tuesday?

Estimate each sum or difference by rounding to 0, \(\frac{1}{2}\), or 1.
30. \(\frac{1}{8} + \frac{4}{7}\)  31. \(\frac{11}{12} - \frac{4}{9}\)  32. \(\frac{4}{5} + \frac{1}{9}\)  33. \(2\frac{9}{10} - 2\frac{1}{7}\)
CUMULATIVE ASSESSMENT, CHAPTERS 1–4

Multiple Choice

1. Which of the following numbers is divisible by 3, 4, and 8?
   - A) 12
   - B) 16
   - C) 20
   - D) 24

2. When June sits down today to read, she notices she is on page 20 of a 200-page book. She decides to read 4 pages of this book every day until she is finished. If this pattern continues, what page of the book will June be on in 10 more days?
   - F) 24
   - G) 44
   - H) 60
   - J) 120

3. Alice is using three different colors of beads to make necklaces. She has 48 blue beads, 56 pink beads, and 32 white beads. She wants to use the same number of pink, same number of blue, and same number of white beads on each necklace. What is the greatest number of necklaces she can make if she uses all of the beads?
   - A) 16
   - B) 12
   - C) 8
   - D) 4

4. A writer spends $144.75 on 5 ink cartridges. Which equation can be used to find the cost c of one ink cartridge?
   - F) 5c = 144.75
   - G) \(\frac{144.75}{5}\)
   - H) 5 + c = 144.75
   - J) 144.75 - c = 5

5. Which fraction is equal to 0.25?
   - A) \(\frac{1}{3}\)
   - B) \(\frac{1}{4}\)
   - C) \(\frac{2}{5}\)
   - D) \(\frac{1}{25}\)

6. Which fraction is NOT equivalent to \(\frac{4}{6}\)?
   - F) \(\frac{2}{3}\)
   - G) \(\frac{10}{15}\)
   - H) \(\frac{8}{12}\)
   - J) \(\frac{16}{18}\)

7. Which fraction is equivalent to the shaded area of the model?

   - A) \(\frac{2}{4}\)
   - B) \(\frac{3}{24}\)
   - C) \(\frac{6}{32}\)
   - D) \(\frac{4}{40}\)

8. Steve bought a movie ticket for $6.25, a box of popcorn for $2.25, and a large drink for $4.75. How much money did he spend at the movie?
   - F) $12.00
   - G) $12.75
   - H) $13.25
   - J) $13.50

9. Four boys each order their own small pizza. William eats \(\frac{3}{5}\) of his pizza. Mike eats \(\frac{2}{5}\) of his pizza. Julio eats \(\frac{1}{2}\) of his pizza. Lee eats \(\frac{3}{8}\) of his pizza. Who ate the least amount of pizza?
   - A) Lee
   - B) Mike
   - C) Julio
   - D) William
10. There are 78 students going on a field trip to the state capitol. The students are in groups of 4. Each group must have an adult leader. How many adult leaders are needed for each student group to have an adult leader?  
   [Options: 15, 19, 20, 22]

11. Which of the following is equivalent to 2.52?  
   [Options: \(\frac{2}{50}\), \(\frac{2}{25}\), \(\frac{2}{10}\), \(\frac{2}{5}\)]

12. What prime number is greater than 90 but less than 100?  

13. Find the least common multiple of 4 and 6.  

14. Suppose you are making fruit baskets that contain 6 bananas, 4 oranges, and 5 apples each. If you need to make 100 fruit baskets, how many apples do you need?  

15. What is the solution to the equation 97.56 + x = 143.07?  

16. The prime factorization of a number is \(2^3 \times 3 \times 5\). What is the number?  

17. The table below shows the number of days it rained each month. How many total days did it rain during the 3-month period?  

<table>
<thead>
<tr>
<th>Month</th>
<th>Rainy Days</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>6</td>
</tr>
<tr>
<td>February</td>
<td>5</td>
</tr>
<tr>
<td>March</td>
<td>7</td>
</tr>
</tbody>
</table>

Short Response

18. Stacie has 16\(\frac{3}{8}\) yards of material. She uses 7\(\frac{1}{8}\) yards for a skirt. How much material does she have left? Write your answer as a mixed number in simplest form. Then give three other equivalent answers, including one decimal.  

19. Maggie says that 348 is divisible by 2, 4, and 8. Is she correct? Give any other number by which 348 is divisible. Explain.  

20. Write the numbers 315 and 225 as products of prime factors. Then list all the factors of each number and find the GCF. Are 315 and 225 prime or composite? Explain.  

21. Suzanne has 317 flyers to mail. Each flyer requires 1 stamp. If she buys books of stamps that contain 20 stamps each, how many books will she need to mail the flyers?  

Extended Response

22. Mr. Peters needs to build a rectangular pig pen 14\(\frac{3}{4}\) meters long and 5\(\frac{1}{2}\) meters wide.  
   a. How much fencing does Mr. Peters need to buy? Show how you found your answer. Write your answer in simplest form.  
   b. Mr. Peters’s pig pen will need 6 meters more fencing than the rectangular pig pen his neighbor is building. Write and solve an equation to find how much fencing his neighbor needs to buy. Show your work.  
   c. If the neighbor’s pig pen is going to be 4 meters wide, how long will it be? Show your work.
Shawnee State Forest

Nestled in the Appalachian foothills of southern Ohio, Shawnee State Forest covers 60,000 acres of wooded hills that are home to colorful wildflowers and various wildlife. Since the state first purchased the land in 1922, the forest has been a popular destination for nature lovers throughout the region.

Choose one or more strategies to solve each problem.

For 1, use the table.

1. The Shawnee State Forest Backpack Trail is a 40-mile loop that begins at the trailhead parking area. The table shows the seven camping areas available along the trail. What is the distance from camp 7 to the trailhead parking area?

2. A hiker on the 40-mile loop stops every 2 miles to take pictures. She also stops every 3 miles to rest. At how many stops along the trail does she both take a picture and rest?

3. Shawnee State Forest began with a land purchase in 1922. The forest is now 12 times its original size. What was the size, in acres, of the original purchase?

4. Two out of every 15 acres of the forest have been set aside and labeled as wilderness. How many acres of wilderness are there?

<table>
<thead>
<tr>
<th>Backpack Trail Distances</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Portion of Trail</strong></td>
</tr>
<tr>
<td>Trailhead to camp 1</td>
</tr>
<tr>
<td>Camp 1 to camp 2</td>
</tr>
<tr>
<td>Camp 2 to camp 3</td>
</tr>
<tr>
<td>Camp 3 to camp 4</td>
</tr>
<tr>
<td>Camp 4 to camp 5</td>
</tr>
<tr>
<td>Camp 5 to camp 6</td>
</tr>
<tr>
<td>Camp 6 to camp 7</td>
</tr>
</tbody>
</table>
Cleveland Metroparks Zoo

Visitors to the Cleveland Metroparks Zoo can view animals in their natural surroundings. Such surroundings include a rain forest, an African savannah, and the Australian outback. The zoo is home to 84 endangered species and the largest collection of primate species in North America. More than 1.2 million people visit the zoo each year.

Choose one or more strategies to solve each problem.

1. A baby female black rhinoceros was born at the zoo in August 2003. The rhino, named Kibibi, weighed 106 pounds at birth and gained about 28.5 pounds per month. About how much did she weigh in December 2005?

2. Of the 600 animal species at the zoo, 3 out of every 40 are primates. How many primate species are there?

For 3, use the table.

3. The table shows average body and tail lengths for some of the primate species at the zoo. Use the information in the table and the clues below to determine which of these species is found in South America.

   • Four of the species are found only in Africa. One of the species is found only in South America.
   • The species with the longest and shortest body lengths come from the same continent.
   • The two species with tails that are longer than 24.5 inches are found in Africa.

<table>
<thead>
<tr>
<th>Primate Species at Cleveland Metroparks Zoo</th>
</tr>
</thead>
<tbody>
<tr>
<td>Species Name</td>
</tr>
<tr>
<td>---------------</td>
</tr>
<tr>
<td>Allen's swamp monkey</td>
</tr>
<tr>
<td>Colobus monkey</td>
</tr>
<tr>
<td>Hamadryas baboon</td>
</tr>
<tr>
<td>Pale-headed saki</td>
</tr>
<tr>
<td>Collared lemur</td>
</tr>
</tbody>
</table>

Problem Solving on Location 223